Empirical M-cycles Balance Investigation on Relationship among the Countries of the Middle East, US and the Extreme Factions

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Abstract

In this paper, we try to detect structural factors of instability and continual conflicts in the Middle East region. We study the problem of m-cycles balance with regard to the countries in the Middle East, extreme factions and the US. The result shows that the unstable m-cycles are dominant in the well known international barrel of gunpowder region. We confirm the quantitative analysis argument with the rigorous conclusions proved in random signed graphs. Our findings in this study suggest that m-cycles balance index might be an effective measure to explore social alliances, and could be used as a measure of relative balance index to see if some network motifs are more stable than others in a society.

Keywords: the Middle East, m-cycles, empirical data analysis, random signed graphs

1. Introduction

Two kinds of tools from mathematics are used in social network analysis to represent information about patterns of ties among social actors: graphs and matrices. Actually, graph and the corresponding matrix can be converted to each other. In this sense, graphs or networks commonly used in sociological field are consisted to represent social relation among entities, or social structure.

Networks are relational structures, and social networks represent structures of dyadic ties between actors. Examples are friendship between individuals, alliances between firms, trade between countries, or international relations among countries. The nature of networks leads to dependence between actors, and also to dependence between network ties. In the case of not causing confusion, graph or network are equivalent to represent the social structure.

Here, we focus on the special settings of networks (graphs), signed graphs, i.e. those social networks with both positive and negative effects. The positive links express trust, like, approval or friendship, whereas negative links indicate distrust, dislike, disapproval or hostile attitudes. In addition, contrasting to former studies, we assign zero to the edges to denote neutral sentiments or attitudes. For example, an actor i hold complicated sentiments for his/her counterpart j, i.e. as to actor i, j is neither his/her friend nor his/her enemy. With the aim of quantitative analysis, more precisely, we use the signs "+" and "–" are assigned to the edges of the corresponding graph in order to distinguish a relation from its opposite, and sign "0" is assigned to the edges to denote the mixed or neutral effects.

As to signed relation, Heider (1946) first presented structure balance theory based on triad, intended to refer to the cognitive structures of an individual person [1]. Heider asserted that a social system was balanced if there was no tension and that unbalanced social structures exhibited a tension resulting in a tendency to change in the direction of balance.

Later, structural balance theory was popularly used to explore social alliances and show that some network motifs are more stable than others in a society. Since the seminal work of Heider, the notion of balance has been extensively studied by many mathematicians and psychologists [2].

Lots of studies show that triadic motif (or 3-cycles) is the basic building block for analyzing global social network properties. Recently, based

on triadic structure balance census, on-line signed social network global property has been studied [3, 4]. Some addressed the inherent connection between network global emerging pattern and local triadic social ties structure balance, where the global pattern refers to an organization, a social group's polarization or stability [5,6].

In 1956, Cartwright and Harary extended the original triads balance or 3-cycles balance (Heider, 1946) to the scenario of m-cycles (the length of m>3) balance [7]. They proposed that the definition of balance with formulation of signed graph may be used generally in describing configurations of many different sorts, such as communication networks, power systems, sociometric structures, systems of orientations, or perhaps neural networks. Their cornerstone result states that a signed graph is balanced if and only if in each cycle (i.e. The closed paths beginning and ending on that same node) contains an even number of negative edges, otherwise is said to be negative.

According to Cartwright and Harary, formally, a signed graph is a graph $G=(V,E)$ together with a mapping $f: E \to \{+,-\}$, which associates each edge with the sign + or −. In such a signed graph, a subset H of $E(G)$ is said to be positive if it contains an even number of negative edges, otherwise is said to be negative. A signed graph G is balanced if each cycle of G is positive. Otherwise it is unbalanced. Their cornerstone result not only states that a signed graph is balanced if and only if in each cycle the number of negative edges is even, but also suggests an approximation of the degree of balance, by investigating the ratio of the number of positive cycles to the total number of cycles in a signed graph.

In order to measure the relative levels of balance on a signed graph, Morissette (1958) introduced the notion of "degree of balance"[8]. Later, Flament, Cartwright and Harary, Taylor and Norman-Roberts suggested a new relative m-balance index. Further, Norman and Roberts proposed to use the ratio of the number of positive cycles of length at most m to the total number of cycles of length at most m to measure the relative m-balance in a signed graph [9]. This idea is described by Formula (1):

$$\rho(m)=\frac{\sum_{m\geq 3} f(m)X_m^+}{\sum_{m\geq 3} f(m)(X_m^+ + X_m^-)} \qquad (1)$$

where X_m^+ (X_m^-) denotes the number of positive (negative) cycles of length m and $f(m)$ is a monotone decreasing function weighting the relative importance of cycles of length m.

In our former studies, we investigated the connection between m-cycles balance and triadic balance through modeling and empirical social networks such as the World War I, and found that 3-cycles or triadic motif balance was consistent with m-cycles ($m>3$) motif balance. In this paper, we study m-cycles on some empirical social networks in real world. The dataset is the relationship among the countries of the Middle East, US and the extreme factions. By using relative m-balance index in a signed graph, we attempt to measure the relative balance level of the well known international barrel of gunpowder region, quantitatively.

2. Quantitative analysis of m-cycles balance on countries in the Middle East, extreme factions and US.

The Middle East lies at the juncture of Eurasia and Africa and of the Mediterranean Sea and the Indian Ocean. It is the birthplace and spiritual center of religions such as Christianity, Islam, Judaism, Manichaeism, etc. Throughout its history the Middle East has been a major center of world affairs; a strategically, economically, politically, culturally, and religiously sensitive area. Five wars had happened there during 1972-1982. In recent years, the unpeaceful situation and emerging extreme factions have aroused the attention of the whole world.

Figure 1 illustrates the complicated national relationships among countries in the Middle East region. Where symbol  denotes friendship relation,  represents hostile relation, and  denotes complicated regional states relations. Obviously ISIS is as enemy for all the countries. One Middle East expert has pointed out that the complicated relationships in this region was better to summarize as "the enemy of my enemy is foe" [10], which is contrary to the classic structural balance theory.

Next, we use Formula (1) to calculate the relative m-balance level for both the empirical da-

tasets which mentioned above. It is easy to transform Figure 1 into adjacent matrix, then we compute the number of m-cycles both positive and negative, where $3 \leq m \leq 10$. In addition, we plot the corresponding m-cycles frequency distribution histogram both positive and negative as in Figure 2, and observe that when m=5,6,7,9,10 the number of balanced m-cycles are more than that of unbalanced counterparts, when m=3,4,8 balanced m-cycles survive together with unbalanced m-cycles. Over all, the instability m-cycles are dominant.

According to Heider (1946) that a social system is balanced if there is no tension and that unbalanced social structures exhibit a tension resulting in a tendency to change in the direction of balance, we might argue that these results could be one of explanations for the continued escalation of the conflict situation of the Middle East tensions. Further, in order to confirm the conflict situations occurred in the Middle East are rooted in the unbalanced m-cycles network structure, we use the rigorous result proved in reference [11]. By extending Heider's and Cartwright-Harary's theory of balance in deterministic social structures, reference [11] studied the problem of balance in social structures with random signed relations between individuals (random signed graph). The main results are suggested as the following two Theorems, the detail proving processes; please refer to [11].

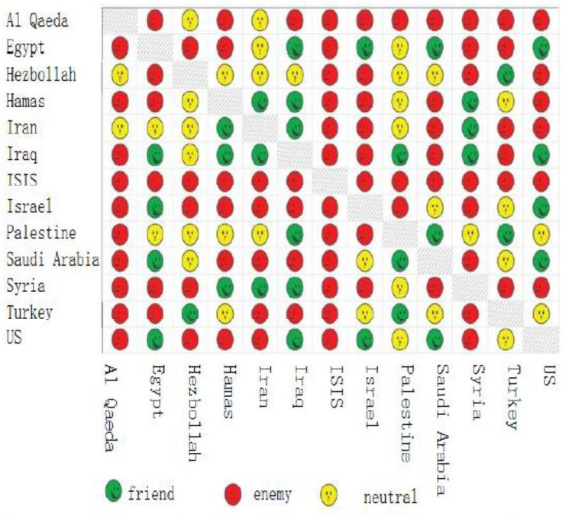

Figure 1. The relationship among the countries of the Middle East, US and the extreme factions. (http://wallstreetcn.com/node/100300)

Theorem 1. Let m be a fixed integer, $0 \leq m \leq n$. Then, almost always in $G_{n,p,q}$, we have $\rho(m) \to \frac{1}{2}$. where $\rho(m) \to \frac{X_m^+}{X_m}$, if $X_m \neq 0$, $\rho(m) \to \frac{1}{2}$, if $X_m = 0$; and $G_{n,p,q}$ denotes random signed graphs, given a set of n vertices between each pair of vertices there exists either a positive edge with probability p, and a negative edge with probability q, or there is no edge with probability $1 - p - q$ connected.

Theorem 2. Let $p = p(n)$ depends on n, if $p >> p_0(n)$, then $G_{n,p,q}$ is unbalanced almost surely, while on the other hand, if $p << p_0(n)$, then $G_{n,p,q}$ is balanced almost surely, where $p_0(n) = 2\log 2 / n$.

Based on Formula (1), we have $\rho(m) = 0.5386$, where $m = 10$, that empirical result is well agreed with the conclusion of Theorem 1. Theoretically, there exists $n(n-1)$ directed edges for a graph with n vertexes, in the Middle East relation network $n = 13$, and there are total 34 positive edges. Since $p = 0.2179,$, and $p_0(n) = 2\log 2 / 13 = 0.0463$, while $p >> p_0(n)$ is satisfied, according to Theorem 2 the Middle East relation graph is unbalanced almost surely.

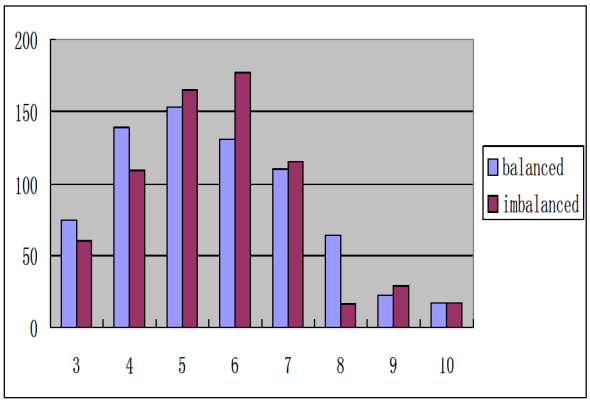

Figure 2. M-cycles frequency distribution

3. Conclusions

The social structure of a population is based on individual social associations, which can be described using network/graph patterns (motifs). Here we use empirical relational data from Middle East, extreme factions and US to see if or not structural balance is present in this traditional

continuous conflicting region. Except for triad or 3-cycles, we focus on long range m-cycles; our computing result shows that the instable m-cycles are dominant in the well known international barrel of gunpowder region. We confirm the quantitative analysis conclusion by using the rigorous results in Random Signed Graphs. Our findings imply that relative m-cycles balance index might be applied as an effective measure to analyze long range social structure stability, from structural sociology aspect, provide a vide a quantitative explanation for this traditional unstable and continuing conflict region.

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