

Group Polarization: Connecting, Influence and Balance, a Simulation Study Based on Hopfield Modeling

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Abstract. In this paper, we address the general question of how negative social influence determines global voting patterns for a group-based population, when individuals face binary decisions. The intrinsic relation between global patterns and local structure motifs distribution is investigated based on Hopfield model. By simulating results with the model, we examine the group opinions polarization processes, and find that the global pattern of group opinions polarization is closely linked with local level structure balance. This computing result is well agreed with the classic structure balance theory of social psychology.

Keywords: global polarization, structure balance, Hopfield model, social influence.

1 Introduction

As one of the important collective actions, global polarization patterns are widely observed in human society. Examples of these situations include culture, social norms, political voting and on-line public hotspots debates. From decision-making perspective, the global pattern of collective dynamics is rooted from individuals' micro-level decision making processes.

The individual decision-making is always as a result of several complex and dynamic social psychological and economic behavior processes. Examples include "herding effect [1]", "wisdom of crowds[2]", "information cascade[3]" etc. Modeling of social system dynamics inevitably involves underlining social psychological processes which have a close connection with real world phenomena. From a bottom-up point of view, in modeling social processes, individuals' locally cumulative interacting behaviors would evolve into different global patterns. The emergence of broad global features of social structure is often from the local interconnecting mechanism, e.g., short average path and high clustering coefficients contribute to small world mechanism [4]. However, it is difficult to infer how the global patterns are generated from certain simple local aggregated social processes. In such cases, agent-based computational techniques are necessary because analytic solutions are simply not available.

In this study, through agent-based Hopfield network simulation, we investigate the underlying relationship between macroscopic group polarization patterns and local dynamic structure balance, which is based on three classic social psychological processes----social influence, social identity and structure balance. Our simulation shows that global polarization of collective voting behavior has the implicit close relation with local dyadic and triadic motifs dynamic structure variation processes. This computing result is well agreed with theories of Heider's cognitive balance [5], Cartwright and Harary generalized structure balance [6].

The rest of the paper is organized as follows: in Section 2, according to social identity theory, we discuss social influence implication and classify the social influences into three types, positive, negative and neutral. Section 3 addresses the classic structure balance theory, and 16 types of triadic classification. Then we define influence structures on the basis of the 16 types of triads. In Section 4, we extend Hopfield model by adding triad social structure. The triad structure implications are also discussed. Section 5 is our model computing analysis. We examine the relationship between negative social influence and polarization, and then focus on the connection between global polarization pattern and dyad/triad balance at the micro-level. Section 6 is our conclusion remarks.

2 Social Influence

Social influence refers to the way people are affected by the thoughts, feelings, and behaviors of others. It is a well-studied and core topic in social psychology. It studies the change in behavior that one affects another, intentionally or unintentionally, as a result of the way the changed person perceives themselves in relationship to the influencer, other people and society in general [7].

In many circumstances, instead of independent rational choice, individual decisions are influenced by their perceptions, observations, or expectations of decisions made by others, and then herding effect might appear. Many theories and studies account for this collective phenomenon. Individuals, for example, may be susceptible to social influence out of a desire to identify with the certain social groups, or to classify oneself from the groups. In order to avoid unexpected sanctions or risks, they may resort to group behavior, or response to influential authorities, as a way of reducing one's decision making difficulty[8].

Recently, an increasing number of empirical studies showed that social networks, such as MySpace, Twitter and Facebook are showing unexpected power, affecting every aspects of our social life, and significantly impacting on individual options, opinions or attitudes as society becomes more inter-connected. The recent "Arab Spring" on Twitter is one of the best annotations [9].

Social influence, therefore, is related both to an individual's cognition of the social world, and to the dynamics of group patterns. Next we classify social influence into three concrete types based on the theory of social identity.

2.1 A Classification of Social Influences

As one of the important theoretical basis in social simulation, social influence mechanism was widely studied. Most social simulation literatures consider the principle

of homogeneous influence (homogenous attraction and ingroup impact), i.e. similarity leads to interaction and interaction leads to more similarity. So the more similar an actor is to a neighbor, the more likely the actor will adopt his/her neighbor's opinion. Individuals blindly follow majority behaviors or options, then the homogeneous influence may create "herding effect", which means individuals don't consider their own subgroup identity (do as most people do). For example, based on the single interaction principle of homogeneous influence, Axelord observed a local convergence and global multiple polarization pattern [10]. And the voter model shows one-polarization domination results with any initial binary opinions percentages [11].

However, the homogeneous impact is not the only social factor to glue individuals together and form different groups. Nation's split, religious conflicts, culture diversity and radical segregation, etc. account for the repulsive attitudes deeply rooted in human collective dynamics. Herding effects only partially explain one aspect of collective behaviors, individuals in a group act together without planned direction, based on indefinite individuals' group recognition. Next, we seek explanation from social identity theory.

Social identity as a basic theory is a powerful tool to predict certain intergroup behaviors on the basis of the perceived status of the intergroup environment. It states that social behavior often varies along a mutual influence processes between interpersonal and intergroup behaviors [12].

Individuals are likely to display favoritism among ingroup and disapproval among outgroup. In other words, individuals usually display positive attitudes toward ingroup members, while negative toward outgroup ones. For example in the case of voting and debating about distribution of national income, different classes may have different interests and political tendencies, individuals often favor ingroup and against outgroup opinions or stands.

In many real situations, negative repulsive impact among social groups is an important ingredient while it has been barely focused together with positive attractive influence behavior in studies. Here, we concern about what the pattern will be if consider both the positive attractive and negative repulsive impact. Next we make a detailed classification for general social influence from social identity perspective.

Positive influence which refers to homogeneous impact among ingroup numbers. Individuals within the same group, share the same tagged consensus, such as beliefs, interests, education or other similar social attributes. During group decision making, homogenous positive influence will play vital role for achieving the group consensus.

The second one is negative social impact which may block the formation of consensus among different outgroups. Individuals within different groups find it difficult to gain the agreement during group decision making even under the pre-condition that they share the same initial opinions. Since different groups have different social group unified interests, emotions, behaviors and value orientations, they act differently. Such kind of impact for individuals' opinions selection is regarded as heterogeneous repulsion. The use of both positive and negative interactions in social systems has been previously introduced to coalitions study among a set of countries [13].

Recently, many studies suggest online social networks in which relationships can be either positive (indicating relations such as trust, friendship) or negative (indicating relations such as opposition, distrust or antagonism). Such a mix of positive and negative links arises in a variety of online settings, e.g., Epinions, Slashdot and Wikipedia [14].

Apart from positive and negative influence, we also observe individuals who seldom care about others and share no commons. As a type of special individuals' attitudes, the individuals might not belong to any labeled subgroup, members in the group have no common social identity, no firm stand about some opinions and are in a state of something else. Then we introduce the third one, unsocial phenomena as a type of special individuals' attitude, in which the individuals do not belong to any labeled subgroup. Members in this group have no common social identity, no firm position about some social opinions and in a state of neither fish nor fowl. According to the above analysis, Figure 1 presents the influence relations among ingroup and outgroup individuals.

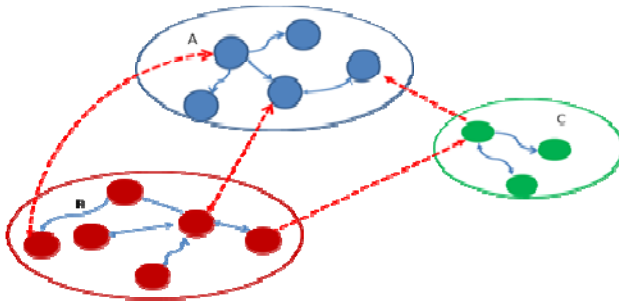


Fig. 1. Three types of social influence in a networked group

Illustration in Fig.1, A, B, C represent three different group. No connection among nodes (individuals) means that they have neutral influence. \longleftrightarrow stands for positive mutual influence among ingroup members, \longrightarrow stands for unilateral positive impact. While $\leftarrow - \rightarrow$ represents mutually negative repulsive among outgroup members, $- - - \rightarrow$ stands for unilateral negative against outgroup individuals.

2.2 Structure Balance

Heider's balance theory is one of the cognitive consistency theories, which addresses the balance on the relationship between three things: the perceiver, another person, and an object[5]. Based on one of Heider's propositions stating that an individual tends to choose balance state in her interpersonal relation, and avoid tension or imbalance state in his/her interpersonal relations. This enforces someone to change her sentiment relation toward balance formation or to lesser force/tension. Cartwright and Harary generalized Heider's cognitive balance to structure balance[6].

Structure balance suggests that some social relationships are more usual and stable than others. It focuses on triadic relationships such as friendship and antagonistic, e.g., graphs whose signed edges represent friendship/hostile relationship among individuals. Structure balance theory affirms that signed social networks tend to be organized so as to avoid tense or nervous situations, based on the common principles that "the friend of my friend is my friend, the enemy of my friend is my enemy, the friend of my enemy is my enemy", this balanced and unbalanced triadic relationship are illustrated in Fig.2. In the illustration, T1 and T2 are balanced triad since the algebraic multiplication of edges signs has a positive value, while T3, T4 are unbalanced.

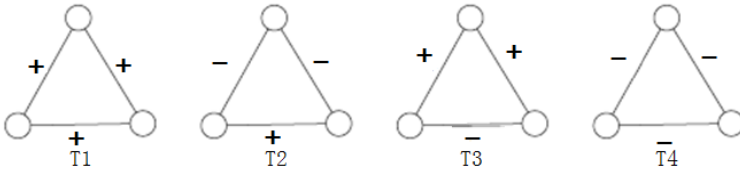


Fig. 2. Balanced and unbalanced triadic relationship (“+” denotes friendship, “-”denotes enemy relation)

Holland and Leinhardt addressed that classic balance theory offers a set of simple local rules for relational change and classified local triadic motifs into 16 types, according to mutual reciprocity, asymmetry relation, non-relationship etc[15]. We can see Code 300 triad relation corresponding to structure balance under the condition of the triad product signs satisfies “+”, as illustrated in Figure 3.

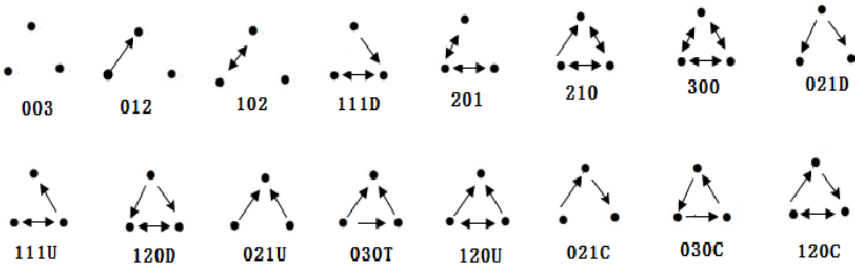


Fig. 3. 16 types of triad distributions in classic structure balance theory

The triad as one of important interpersonal relation has been excessively investigated by sociologists, e.g., Watt and Strogatz suggested that clustering coefficient is one of the important small world local structure features [16]. Wasserman and Faust defined a global clustering coefficient to indicate the clustering in the whole network[17]. In the next section, we discuss the influence balance on triad and dyad.

2.3 Influence Balance on Dyad and Triad

For any individual in a group, his/her options adoption comes from the influence around him/her and the corresponding cumulative social pressure. For example, if one’s main friends have bought the same brand mobile phone, one might have high possibility to buy a mobile phone with this brand.

Here we focus on triadic relation influence balance. In Fig.3, we use directed edge represents directed influence relation. For example, the interpersonal influence on triad 111D includes three dependent dyadic social influence structure, in detail $i \rightarrow k$ stands for i has positive influence on k , $j \leftrightarrow k$ stands for j, k have mutual negative influence and no connection represents i, j have neutral influence as illustrated in Fig. 4.

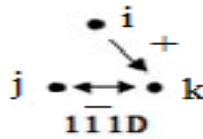


Fig. 4. The influence relation on code 111 D

There exist two basic structures that could satisfy influence balance in 16 motifs. For dyadic influence relation, Code 120 stands for mutual positive or negative influence. For triadic influence relation Code 300 is balanced if and only if product of any mutual influence relation signs satisfies +.

Next we examine Hopfield network model based on the aforementioned three types of social influence mechanism. We focus on two aspects, one is negative impact on group voting stable pattern, and the other is the relation between global pattern and local dyad, triad distribution. This model is described in the following section.

3 Hopfield Network Model

Macy et al. presented a Hopfield model to describe group polarization problems, with continuous connection weights and principles of homogeneous attraction and heterogeneous repulsion [18]. Their study found that group can display consensus, bipolarization and pluralistic alignments under different social pressures and exterior interventions. Their model assumed that each individual face binary opinions and has $N-1$ undirected ties to others. These ties measured by weights, which determine the strength and valance of connection between agents. Formally, social pressure P_{is} on individual i to adopt a binary state $s_i = \pm 1$ is the sum of the states of all other individuals j , conditioned by the weights I_{ij} ($-1 < I_{ij} < 1$) of dyadic tie between i and j :

$$P_{is} = \frac{\sum_{j=1}^N I_{ij} s_j}{N - 1}, j \neq i, \tag{1}$$

If consider the external intervention, i.e., the influence for individuals opinion comes from other out-group impact, then we can replace Equ.(1) with Equ. (2) and obtain the logistic form

$$\tau_{is} = \frac{v_s}{1 + e^{-\kappa P_{is}}} + (1 - v_s) X_i. \tag{2}$$

where v_s is used to trade off the internal and external group influence for individual i opinion, κ is the size of opinions dimension. Given a randomly selected threshold $\pi_{thresh} = 0.5 + \epsilon\chi$, if $\tau_{is} \geq \pi_{thresh}$, individual i chooses +1 (support), else chooses -1 (oppose), where ϵ is Harsanyi smooth responding parameter χ is subject to uniform distribution between -0.5 and +0.5. Equ. (3) describes the update of influence processes of individual j to i ($j \neq i$).

$$I_{ij}(t + 1) = I_{ij}(t)(1 - \lambda) + \frac{\lambda}{K} \sum_{k=1}^K s_{jk}(t)s_{ik}(t), j \neq i, \tag{3}$$

where t is the time step, λ is an adjustable parameter between 0 and 1. Comparing with [18], we extend the original Hopfield model from two aspects.

Firstly, with the motivation of investigating the relationship between non-positive social influence and group opinions polarization, instead the assigning of continuous values between -1 and $+1$ to I_{ij} , we assign three discrete values $-1, +1, 0$ to I_{ij} to indicate the three types of social influence. Individuals (agents) are influenced by others and also influence others, as conditioned by the valence of the social identity tie I_{ij} , where $I_{ij} = \{+1, 0, -1\}$ listed and explained as follows:

- 1) “+1” denotes the positive homogeneous social influence,
- 2) “-1” stands for xenophobia, antagonistic, negative social influence,
- 3) “0” represents neutral influence.

Secondly, Equ.(1) shows that individual i cumulative social pressure is from dyadic structure, and the triadic influence relation and corresponding cumulative social pressure is not included in the Hopfield model. Here, we add triadic influence into the model. Equ.(1) is evolved into Equ.(4) as following form,

$$P_{is} = \frac{\sum_{j=1}^N I_{ij} s_j}{N - 1} + \frac{2 \sum_{j \neq m \neq i=1}^N I_{ijm} s_j s_m}{(N - 1)(N - 2)}, j \neq i. \tag{4}$$

where the second term represents the cumulative social pressure that any two individuals j, m impose on i in triadic structure, simultaneously. $I_{ijm} = \{+1, 0, -1\}$ stands for the social influence that j, m impose on i . With triad influence structures included in the model, it can better describe the real world interpersonal dynamic processes and opinions formation. For example, if we only consider dyadic influence, the cumulative social pressure of any individual i just depends on his/her in-degree d_i . However, with triadic influence included, one part of the social pressure will come from local clustering coefficient C_i . Since we put triad influence structure included in Equ.(4), the corresponding triadic social influence weight update processes is described with Equ.(5),

$$I_{ijm}(t + 1) = I_{ijm}(t)(1 - \lambda) + \frac{\lambda}{K} \sum_{k=1}^K s_{jk}(t)s_{ik}(t)s_{mk}(t), j \neq i. \tag{5}$$

As in Equ.(3), λ is an adjustable parameter between 0 and 1. Here we name λ is the social influence evolutionary parameter, which is used to adjust social influence strength variation.

Next, we will focus on the intrinsic relation between group polarization and social influence from both dyads and triads based on the extended Hopfield model simulation. The pseudo code is listed as follows.

4 Simulation and Results Analysis

In this section, firstly we use Matlab computing platform to simulate group voting polarization processes, then detect local dyad, triad distribution by R package sna[19].

4.1 Simulating Procedure

The pseudo codes of implementing Hopfield network social influence processes are listed as below:

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Step 1: Let  $t = 0$ , given  $v_s, \lambda, \varepsilon$ 
        Initialize each voter  $k$ -th dimension options
         $s_{ik}(0) = \pm 1, k = 1, \dots, K; i, j, m = 1, \dots, N$ , Randomly generate each
        pair of voters' dyadic and triadic social influence  $I_{ij}, I_{ijm}$ , respectively.

Step 2:  $t = t + 1$ , compute (4), (2) for each agent  $i$ 
        Randomly generate  $\chi \sim U(-0.5, 0.5)$ , then compute
         $\pi_{thresh} = 0.5 + \varepsilon \chi$  and logistic social pressure  $\tau_{is}$ 
        if  $\tau_{is} > \pi_{thresh}$ 
             $s_{ik}(t) = 1$ 
        else
             $s_{ik}(t) = -1$ 

Step 3: For a given small positive real number  $\delta$ , compute
        (3), (5)
        If  $\max(|I_{ij}(t) - I_{ij}(t-1)| \cup |I_{ijm}(t) - I_{ijm}(t-1)|) < \delta$ 
            stop
        else
            go to Step 2.
    
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4.2 Negative Social Influence Promotes Group Bi-polarization

We take the test by setting $N = 100, K = 5, \varepsilon = 0.01$ and $\lambda = 0.5$. To illustrate the group voting pattern, we generate $N \times K$ matrix which N denotes for group size, K for the number of options. We run 100 times for average. Fig.5 shows the group initial random opinions states when each agent i faces K -dimension options (before group polarization). Fig.6 illustrates the group bi-polarization state under the condition of no imposing external influence ($v_s = 1$) and with three types of influence. We can observe that two patterns appear after group polarization, i.e., one pattern is $(-1, -1, -1, -1, -1)$, i.e., (black, black, black, black, black) (marked by V_1), the other is $(+1, +1, +1, +1, +1)$, i.e., (white, white, white, white, white) (represented by V_2). The ratio of the 2-pattern size is approximate to 1:1.

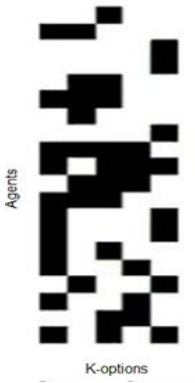


Fig. 5. Initialization of group options



Fig. 6. Bi-polarization pattern of group options

The relationship between exogenous intervention parameter v_s to group polarization is shown in Figure 7. We can see that when $v_s = 1$ (no external intervention to the group interaction processes), the ratio of $V_1 / (V_1 + V_2)$ is approximate to 0.5. However, the fifty to fifty well matched equilibrium will be destroyed with a little cut off v_s . In other words, external intervention will lead to majority pattern appeared. For example, when $v_s = 0.9$ we observe the group consensus appears, i.e., $V_1 / (V_1 + V_2)$ is approximate to 1, the pattern V_2 nearly disappears. It is worth pointing out that the parameters \mathcal{E}, λ variation also impacts the group polarization patterns; however, exogenous intervention is the dominant factor that leads to group polarization.

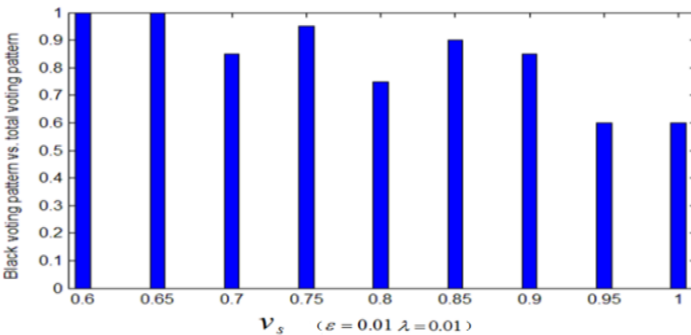


Fig. 7. v_s impact on group opinions polarization

4.3 Dyad and Triad Distribution Before and After Bi-polarization

Furthermore, we also investigate the triadic relation motifs distribution before and after bi-polarization by using R package sna. We find that the overwhelming structure balance motifs emerge and concurrently with the polarization process.

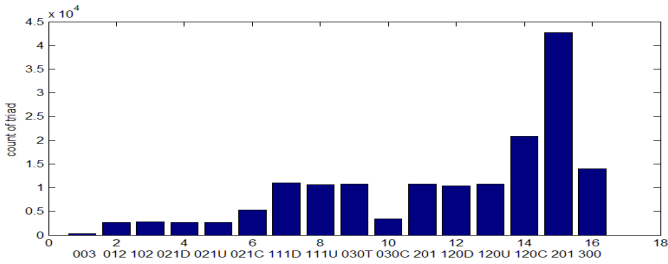


Fig. 8. Initial triad distribution before bi-polarization of group opinions

Fig.8 shows the initial local triads distribution according to randomly generated social influence matrix ($t=0$). We can observe that all 16 types of triads exist in the initial triadic relationships. With the social influence matrix updating and individuals' option changes, group voting bi-polarization emerges at step $t = 30$. Corresponding triad distribution as illustrated in Fig. 9, we observe that other triads disappear and only triad Code 300 remains. We also find the algebraic multiplication of influence signs in all Code 300 relation has a positive value. This result suggests social influence balance emerges from interpersonal negative or positive influence relations among agents. Simultaneously, neutral influence relation disappears.

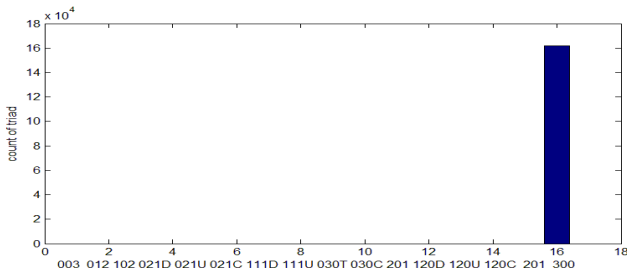


Fig. 9. The triad distribution after bi-polarization of group opinions (code 300 corresponding to the structure balance triad)

We also investigate the dyadic influence relations distribution before and after group bi-polarization. Fig.10 shows the initial local dyad distribution according to randomly generated social influence matrix ($t=0$). We can observe that three types of dyads exist in the initial triadic relationships. Asymmetrical dyadic influence relation is dominant among the three types.

When the global voting pattern of bi-polarization reaches, the corresponding dyad distribution is illustrated in Fig. 11. We find that asymmetrical and neutral influence relations between each individual disappear. Only mutual influence relation (mutual positive or negative impact) remains.

According to above computing results analysis, we observe that dyadic and triadic influence balance among agents has inherent relation with global bi-polarization pattern. This is similar to the macro-micro linkage: sentiment relations among agents (localized as triad and dyad) lead to the collective balance of the group. In other words, we observe the micro foundation (at dyadic/triadic level) of the collective global pattern.

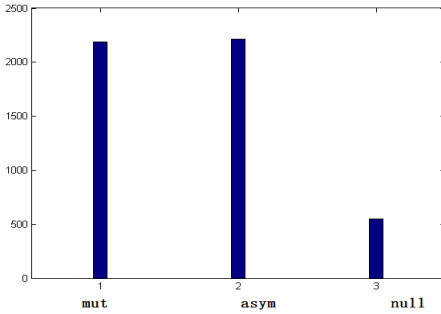


Fig. 10. Initial dyad distribution before bi-polarization of group opinions

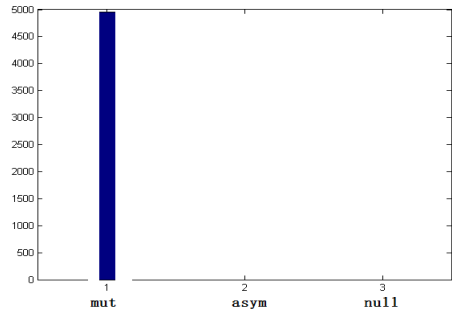


Fig. 11. Final dyad distribution after bi-polarization of group opinions

5 Conclusion

In this paper, we address the implications of three types of social influence based on social identity theory. We investigate the non-positive social impact on group polarization based on Hopfield network model with both dyad and triad influence considered. By simulation we find that bi-polarization pattern tends to emerge with no imposing external intervention, and consensus may occur among group members if the non-positive influence is neglected.

Most literatures suggest that the homogeneous social influence will bring the global stability of social homogeneity, where convergence to one leading polarization is almost irresistible in a closely interconnected or interrelated population. However, in this paper the simulation based on Hopfield network model demonstrates that social homogeneous stable state is highly brittle if “influence ties” are either to be negative or zero. The result argues that bi-polarization may also be attributed to in-group/out-group differentiation and rejection antagonism, which conclusion is consistent with our former study [20].

It is worth pointing out that individuals can evolve into local dyadic and triadic balanced states after group opinion polarization. This conclusion shows that opinions polarization in a group is coexisted with local level structure balance, which reveals some interesting internal connection between global collective pattern and local social structure stability.

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