

# Group Polarization and Non-positive Social Influence: A Revised Voter Model Study

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**Abstract.** In this paper, we analyze how the non-positive social influence affects group polarization by adding influence factor into the classic voter model. Through model simulation, we observe that a group would self-organize into two-polarization pattern, under no imposing intervention, which is entirely different from the result of drift to an extreme polarization dominant state in the classic voter model.

**Keywords:** group polarization, non-positive social influence, social identity, voter model, opinions dynamics.

## 1 Introduction

Research on collective emergence patterns has a long history [1,2]. It is widely studied in management science [3], social psychology [4, 5, 6], economics [7], socio-physics [8], system science [9] and computer science [10] etc. Recently, with the booming of Internet and information technology, especially when it entered into the Web 2.0 era, more and more collective behaviors are observed via Web 2.0 tools, which also provides great opportunities and challenges for this research. With this background, now the topic is maturing into the spotlight for social psychology, social risk emergency management, computer science, marketing and nearly all aspects of Web-based application or online emerging collective behaviors.

In this paper, we focus on one of the important aspects of human collective behaviors—opinions dynamics. We study what's the role that the three kinds of social influence would play for the group polarization, and the intrinsic relationship between three kinds of social influence factors and the group polarization. The rest of the paper is organized as follows: in section II, according to social identity, we discuss three kinds of social influence implication. In section III, we add three types of social influence into the classic voter model. By simulation computing we find an interesting conclusion that a group opinions with binary states could evolve into two-polarization steady pattern. Section IV is our conclusion remark.

## 2 Three Types of Social Influence

Social influence refers to the way people are affected by the thoughts, feelings, and behaviors of others. Like the study of attitudes, it is a traditional, core topic in social psychology. It studies the change in behavior that one person causes in another, intentionally or unintentionally, as a result of the way the changed person perceives themselves in relationship to the influencer, other people and society in general [11]. Social influence occurs when an individual's thoughts, feelings or actions are affected by other people. Social influence takes many forms and can be seen in conformity, socialization, peer pressure, obedience, leadership, persuasion, sales, and marketing. In 1958, Harvard psychologist, Herbert Kelman identified three broad varieties of social influence [12]. Latter development contributors mainly include French (1956) [4], Latané, B. (1981) [5], and Friedkin, N. (1998) [6].

Social influence theory is one important theoretical basis in social science study. Most social simulation literatures consider the principle of homogeneous influence (attraction and social influence, the principle is similarity leads to interaction, and interaction leads to still more similarity). The basic premise is that the more similar an actor is to a neighbor, the more likely that the actor will adopt one of the neighbor's opinion. The homogeneous influence could be considered as "herd behavior" [19] or "information cascade" [20], which means individuals don't consider their own subgroup identity ( "do as most people do" ). For example, based on the single interaction principle of homogeneous influence, Axelord [13] observed a local convergence and global multiple polarization pattern, the voter model show one polarization opinions dominant result with any initial binary opinions percentage [14,15,16].

However, in respect of mutual influences, heterogeneity repulsion and unsocial attitude can not be ignored in a social system. In other words, individuals' opinions not only depend on homogeneous similarity, but also are influenced by heterogeneous repulsion and unsocial attitudes. We may refer to these three types of influence as "social identity" [17], which assumes that individuals have their own social identity that may belong to different specified social community or tagged social subgroup. Individuals within the same subgroup, share the same tagged consensus, such as beliefs, interests, education or other similar social attributes [18]. Since they share the common social tag (identity), when they face group decision making, homogenous positive influence will play vital role for achieving the group consensus. In this paper, in order to classify different influence factors, this kind of within group homogenous impact is called homogeneous influence.

Individuals within different subgroups find it difficult to gain the agreement when they face group decision making even under the pre-condition that they share the same initial opinions, since they have different social subgroup unified interests, emotions, actions and value orientation. This impact for individuals opinions selection can be named heterogeneous repulsion. The third one, unsocial phenomena is a type of special individuals attitude, in which the individuals do not belong to any tagged subgroup. Members in this group have no common

social identity, no firm position about some social opinions and in a state of neither fish nor fowl.

### 3 Description of The Model

#### 3.1 Classic Voter Model

The voter model [14,15,16] is a simple mathematical model of opinion formation in which voters are located at the nodes of a network, each voter (individual) has an opinion (in the simplest case, -1 or +1), for a randomly chosen voter, its opinion have the chance of being affected by the opinions of its neighbors. It is often used to see how ordered states can appear in systems originally in a state of non-equilibrium. This has several applications in a variety of disciplines including chemistry ( reactions between different chemicals ), physics ( interactions between particles ) and social systems ( interactions between agents ). The model formula is shown as Equ (1)

$$\varpi(-\sigma_i|\sigma_i) = \frac{\beta}{2} \left(1 - \frac{1}{k} \sum_{j \in n(i)} \sigma_i \sigma_j\right). \quad (1)$$

Where  $\beta$  is a constant adjustable parameter,  $\sigma_i = +1, -1 (i = 1, \dots, N)$ ,  $N$  is the group size, voter  $i$  opinion “+1” means “for”, “-1 ” means “against”,  $n(i)$  denotes for voter  $i$ 's neighbors,  $k$  is the neighbors size. The left term of Equ (1) is the probability that individual  $i$  might change it's opinion from  $\pm 1$  to  $\mp 1$ . There is a large number of literatures on the relationship between voter model and different topologies, such as scale-free, small world, lattices [16,21,22]. Especially in lattices, the voter model presents simple non-equilibrium dynamics with nontrivial behavior, hence this model has been extensively studied, most notably by the mathematicians and condensed matter physicists, from various aspects.

#### 3.2 Voter Model with Three Types of Social Influence

In order to further understand the relationship between non-positive social influence and group opinions polarization quantitatively, we adopt three kinds of social influence mechanism on the classic voter model (see Equ (2)) in the topology of lattices. Individuals (voters) are impacted by others and also influence others, as conditioned by valence of the social identity tie:

- (A) “+” implies attraction (homophily, similarity) and imitation,
- (B) “-” stands for xenophobia (heterophily instead of homophily) and differentiation ( instead of imitation),
- (C) “0” denotes for unsocial attitudes.

$$\varpi(-\sigma_i|\sigma_i) = \frac{\beta}{2} \left(1 - \frac{1}{k} \sum_{j \in n(i)} I_{ij} \sigma_i \sigma_j\right), \quad (2)$$

where  $I_{ij} \in \{-, 0, +\}$ . To illustrate how the revised voter model works, an example is given: voter  $i$  has the opinion  $+1$  while its eight neighbors are all of opinion  $-1$ . By setting these values into the equation, considering different social influence cases, when all  $I_{ij} = -$ , means agent  $i$  and its neighbors have no common social identity, the probability that it will change its initial opinion state is 0. When all  $I_{ij} = +$ , means agent  $i$  and its neighbors share the common social identity, the probability that it will change its initial opinion state is 1. when  $I_{ij} = +, -$  half half or all equal to 0, means agent  $i$  and its neighbors share half of the common social identity or no firm position, the probability that it will change its initial opinion state is  $1/2$ . If we don't consider the social identity, we can see that the probability of voter  $i$  switching to the opposite state is always equal to 1, this particular case means  $I_{ij} = +$ , and corresponding to the single homogeneous influence scenarios, the classic voter model. Next, through simulation, we would like to find out what result come out if we consider the three types of social influences on the classic voter model in a closed lattice community.

### 3.3 Simulation Implementation

The revised voter model simulation is implemented as the followings pseudocode: Fix  $\beta$ ,  $T$  (final step of simulation) and group size  $N$ .

Step I: initial each voter  $i$ 's opinion state, and social influences matrix  $I$   
for  $i = 1 : N$

initialize  $\sigma_i = \pm 1$ ;

for  $j = 1 : N$

initialize  $I_{ij} = +1, 0, -1$ ;

end

end

Step II: compute Equ (2), obtain voter  $i$ 's opinion possible commutable probability  $Pr_{value}$ ;

Step III: update voter  $i$  opinion state at each time step  $t$

for  $t = 1 : T$

given threshold  $\tau_I = rand()$ ;

if  $Pr_{value} > \tau_I$  and  $\sigma_i = \mp 1$

$\sigma_i = \pm 1$ ;

else if  $Pr_{value} > \tau_I$  and  $\sigma_i = \pm 1$

$\sigma_i = \mp 1$ ;

end

if  $Pr_{value} \leq \tau_I$  and  $\sigma_i = \mp 1$

$\sigma_i = \mp 1$ ;

else if  $Pr_{value} \leq \tau_I$  and  $\sigma_i = \pm 1$

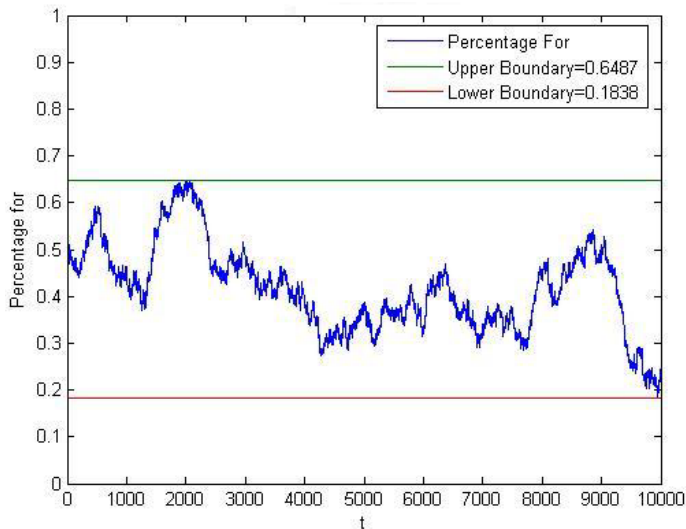
$\sigma_i = \pm 1$ ;

end

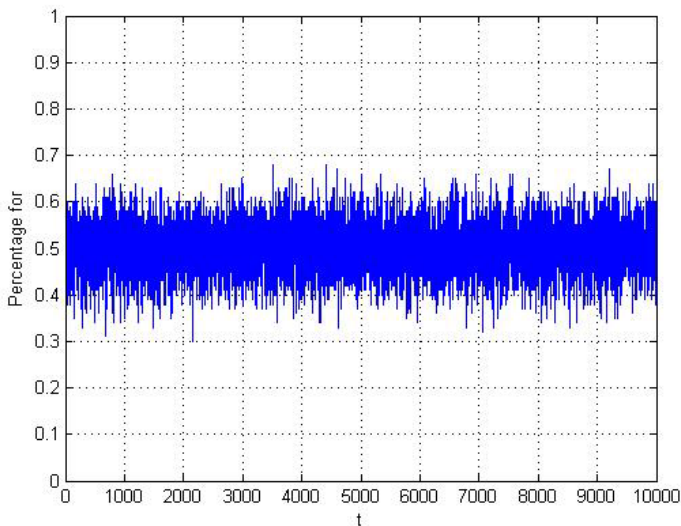
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### 3.4 Simulation and Discussion

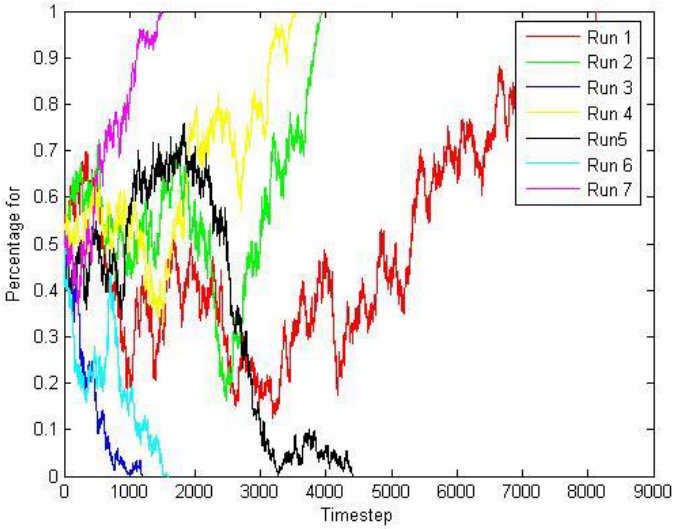
In our simulation we set  $\beta = 1$ , the neighbor size  $k = 8$ , the topology is  $100 \times 100$  periodic boundaries lattices,  $T = 10^4$ . Fig. 1 show that the classic voter model final “+1” percentage fluctuation against time step  $t$ , in the end, it is



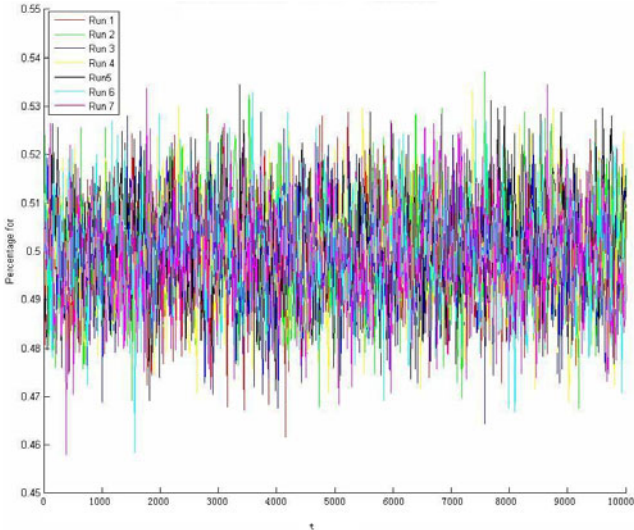
**Fig. 1.** Classic voter model one polarization dominant state with time evolution (50% for or “+1” initialization)



**Fig. 2.** Revised voter model two polarization steady state against time  $t$  (10% for or “+1” initialization)



**Fig. 3.** Classic voter model one polarization dominant trend. Percentage for “for” variation in 7 times runs. Each of the runs was initialized under the same conditions (50% for initialization).



**Fig. 4.** Revised voter model opinions dynamics two polarization steady states. Percentage for “+1” variation in 7 times runs. Each of the runs was initiated under the same conditions (50% for initialization).

prone to reach one polarization steady state even with 50% “+1” initialization. Fig. 2 show the revised voter model with 3 kinds of social influence final “+1” percentage tend to 50%, although in the case with initial 10% “+1”.

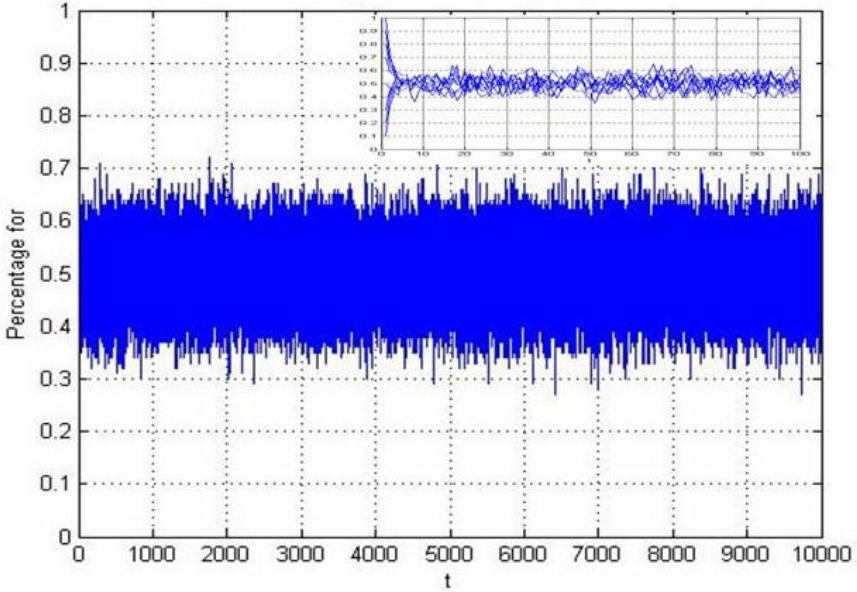


Fig. 5. Population for over time for 10 different initial for percentage

Further model analysis as shown in Fig.3, even with the initial percentage for “+1” is 50%, multiple runs still saw the classic voter model reach an “absorbing” or one polarization dominant state. On the contrary, we observe the two polarization opinions well-matched steady equilibrium in Fig. 4, that interesting result we can not obtain in the original voter model.

In addition, Fig. 5 shows percentage “for” (state=+1) variation in 10 times runs. Each of the runs was initialized under the conditions of “for” from 10% to 100%. The inset plot show that two antagonistic well matched cliques appear easily and convergent to stable soon. This result illustrate that even at a very low/high “for” percentage the revised voter model still reverse direction and move back to a steady state of two-polarization equilibrium. moreover the convergent rate of reaching two polarization steady equilibrium is very soon, nearly within 10 time steps. That important simulation result tell us the social homogeneity is highly brittle.

With heterogeneous exclusion ( “-” ) and unsocial ( “0” ) factors into the classic voter model, we clearly see that a group opinions homogeneous consensus could not be realized, except the non-positive repulsion impact is eliminated.

## 4 Conclusions

In this paper, we consider three kinds of social influences implication based on social identity theory. Especially, after we add the three types of social influences

into the classic voter model, simulation observes one fascinating result: a state of two-polarization equilibrium will appear soon even at a very low/high percentage “for”, which is completely different from the result of drift to a single “+1/-1” extreme one polarization dominant state in the classic voter model. It is also shown that the consensus could not occur with considering the non-positive social influence as does on regular two-dimensional lattices, instead the system settles in a stationary state with coexisting opinions two-polarization. This result also well agrees with the conclusion drawn by Castellano et al[16], and is consistent with the earlier work on structural balance [23].

The original voter model emphasized the global stability of social homogeneity, where convergence to one leading polarization is almost irresistible in closely interacting populations. However, the voter model with some “influence ties” to be negative or zero suggests that social homogeneous stable state is highly brittle.

This study also demonstrates that in-group/out-group differentiation and rejection antagonism are the emergent properties of social network self-organization, and are labelled in the voters’ cognitive architectures as assumed by social identity theory, our argument is different with the conclusion that agents’ cognitive are not inscribed in Macy’s work[4] .

We contribute to this literature by looking into “facet” of self-identity of group members. Our findings indicate that the voting behavior of heterogeneous group is, in fact, different from that of homogeneous. The prism of social identity theory, which holds that people maintain an “us” versus “them” portrait during the processes of the collective behaviors is the explanation of heterogeneous group voting result.

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