2010 IEEE/WIC/ACM International Conference on Web Intelligence and Intelligent Agent Technology

Possibility Degree Method for Ranking Intuitionistic Fuzzy Numbers

Cui-Ping WEI
College of Operations Research and Management
Qufu Normal University
Shandong 276826, P.R.China
Email: wei_cuiping@yahoo.com.cn

Xijin TANG
Academy of Mathematics and Systems Science
Chinese Academy of Sciences
Beijing 100190, P.R.China
Email: xjtang@iss.ac.cn

***Abstract*—In this paper, we study the method of ranking intuitionistic fuzzy numbers. Firstly a possibility degree formula is defined to compare two intuitionistic fuzzy numbers. We prove the ranking order of two intuitionistic fuzzy numbers obtained by the possibility degree formula is the same as the one by using the score function defined by Chen and Tan. Moreover, the possibility degree formula can provide additional information for the comparison of two intuitionistic fuzzy numbers. Based on the possibility degree formula, we give a possibility degree method for ranking n intuitionistic fuzzy numbers and then to rank the alternatives in multi-critera decision making problems.**

***Keywords*-multi-attribute decision making; intuitionistic fuzzy number; possibility degree method**

I. INTRODUCTION

Atanassov[1] introduced the concept of an intuitionistic fuzzy set(IFS) characterized by a membership function and a non-membership function. Gau and Buehrer[3] introduced the concept of vague sets. Bustince and Burillo[2] showed that vague sets are IFSs. IFSs have been found to be more useful to deal with vagueness and uncertainty problems than fuzzy sets, and have been applied to many different fields.

For the fuzzy multiple criteria decision making (MCDM) problems, the degree of satisfiability and non-satisfiability of each alternative with respect to a set of criteria is often represented by an intuitionistic fuzzy number(IFN), which is an element of an IFS[4],[5]. The comparison between alternatives is equivalent to the comparison of IFNs. Chen and Tan[6] provided a score function to compare IFNs. Hong and Choi[7] pointed out the defects and proposed an improved technique based on the score function and accuracy function. Later, Li and Liu[8],[9] gave a series of improved score functions. Both score function and accuracy function are called the evaluation functions. By using these evaluation functions, we can obtain the certain ranking of the IFNs. Since IFNs are of fuzziness, the comparison between them may also be expected to reflect the uncertainty of ranking objectively .

In this paper, by extending the possibility degree method of interval-valued numbers[10],[11] to intuitionistic fuzzy sets, we propose a possibility degree method for ranking IFNs. We prove the same ranking can be reached by using both the possibility degree method and the score function defined by Chen and Tan. And the ranking result by the possibility degree method may reflect the uncertainty of IFSs, and then provide more information to decision makers.

II. POSSIBILITY DEGREE METHOD FOR RANKING INTUITIONISTIC FUZZY NUMBERS

A. Possibility degree formula for ranking two intuitionistic fuzzy numbers and its properties

Let I=[0,1],$\vee$=max,$\wedge$=min.

Definition 2.1[1] Let X be an ordinary finite non-empty set. An intuitionistic fuzzy set on X is an expression given by $A = \{\langle x, u_A(x), v_A(x)\rangle | x \in X\}$, where $u_A : X \to I, v_A : X \to I$ with the condition $0 \le u_A(x) + v_A(x) \le 1$ for all $x \in X$. $u_A(x)$ and $v_A(x)$ denote, respectively, the membership degree and the nonmembership degree of the element $x \in A$. We abbreviate "intuitionistic fuzzy set" to IFS and represent IFS(X) the set of all the IFS on X. We call $\pi_A(x) = 1 - u_A(x) - v_A(x)$ the degree of hesitation (or uncertainty) associated with the membership of element x in A.

According to the research in [4][5], for an IFS $A = \{(u_A(x), v_A(x))|x \in X\}$, the pairs $(u_A(x), v_A(x))$ is called an intuitionistic fuzzy number (IFN). For convenience we denote an IFN by (a, b), where $a \in I, b \in I, a + b \le 1$. Let Q be the set of all the IFNs.

Definition 2.2[5] Let $\alpha_i = (a_i, b_i) \in Q, i = 1, 2$, then

1) $(a_1, b_1) = (a_2, b_2) \Leftrightarrow a_1 = a_2, b_1 = b_2$;
2) $(a_1, b_1) \ge (a_2, b_2) \Leftrightarrow a_1 \ge a_2, b_1 \le b_2$;
3) $(a_1, b_1) > (a_2, b_2) \Leftrightarrow a_1 \ge a_2, b_1 \le b_2$ & $(a_1, b_1) \ne (a_2, b_2)$;
4) $\overline{\alpha}_1 = (b_1, a_1)$.
5) $\alpha_1 + \alpha_2 = (a_1 + a_2 - a_1a_2, b_1b_2)$;
6) $\alpha_1\alpha_2 = (a_1a_2, b_1 + b_2 - b_1b_2)$;
7) $\lambda\alpha_1 = (1 - (1 - a_1)^\lambda, b_1^\lambda), \lambda > 0$;
8) $\alpha_1^\lambda = (a_1^\lambda, 1 - (1 - b_1)^\lambda), \lambda > 0$.

978-0-7695-4191-4/10 $26.00 © 2010 IEEE
DOI 10.1109/WI-IAT.2010.239

For the practical MCDM problems, experts need to obtain the rank of the alternatives. Suppose the comprehensive evaluation value of each alternative is represented by an IFN α, where $\alpha = (a, b)$, which indicates the degree of satisfiability and non-satisfiability of each alternative with respect to all the attributes. The larger the degree of hesitation $\pi(\alpha)$, which is equal to $1 - a - b$, the bigger the possible change scope of the degree of satisfiability and non-satisfiability of the alternative for the experts. As the comprehensive evaluation value is denoted by (a, b), the degree of satisfiability of the alternative for the experts is actually an interval value written as $[a, a + \pi(\alpha)]$. Similarly, the degree of non-satisfiability of the alternative for the experts can be written as $[b, b + \pi(\alpha)]$. Therefore, the comparison between IFNs can be solved by using the possibility degree formula of interval values. Next we extend the possibility degree method of interval-valued numbers[10],[11] to intuitionistic fuzzy sets and define a possibility degree formula to compare two IFNs.

Definition 2.3 Let $\alpha_1 = (a_1, b_1)$, $\alpha_2 = (a_2, b_2)$, $\pi(\alpha_1) = 1 - a_1 - b_1$, $\pi(\alpha_2) = 1 - a_2 - b_2$. If $\pi(\alpha_1) = \pi(\alpha_2) = 0$, we call

$$p(\alpha_1 > \alpha_2) = \begin{cases} 1, & a_1 > a_2, \\ 0, & a_1 < a_2, \\ \frac{1}{2}, & a_1 = a_2, \end{cases}$$

the possibility degree of $\alpha_1 > \alpha_2$.

Definition 2.4 Let $\alpha_1 = (a_1, b_1)$, $\alpha_2 = (a_2, b_2)$, $\pi(\alpha_1) = 1 - a_1 - b_1$, $\pi(\alpha_2) = 1 - a_2 - b_2$. If $\pi(\alpha_1), \pi(\alpha_2)$ is not zero at the same time, we call

$$p(\alpha_1 > \alpha_2) = \frac{max\{0, (a_1 + \pi(\alpha_1)) - a_2\} - max\{0, a_1 - (a_2 + \pi(\alpha_2))\}}{\pi(\alpha_1) + \pi(\alpha_2)}$$

the possibility degree of $\alpha_1 > \alpha_2$.

Definition 2.5 If $p(\alpha_1 > \alpha_2) > p(\alpha_2 > \alpha_1)$, then α_1 is superior to α_2 with the degree of $p(\alpha_1 > \alpha_2)$, denoted by $\alpha_1 \overset{p(\alpha_1>\alpha_2)}{\succ} \alpha_2$; If $p(\alpha_1 > \alpha_2) = p(\alpha_2 > \alpha_1) = 0.5$, then α_1 is indifferent with α_2, denoted by $\alpha_1 \sim \alpha_2$;

If $p(\alpha_2 > \alpha_1) > p(\alpha_1 > \alpha_2)$, then α_1 is inferior to α_2 with the degree of $p(\alpha_2 > \alpha_1)$, denoted by $\alpha_1 \overset{p(\alpha_2>\alpha_1)}{\prec} \alpha_2$.

It is easy to prove that:

Theorem 2.1 Let $\alpha_1 = (a_1, b_1)$, $\alpha_2 = (a_2, b_2)$, then

(1) $0 \le p(\alpha_1 > \alpha_2) \le 1$;

(2) $p(\alpha_1 > \alpha_2) = 1 \Leftrightarrow a_1 \ge a_2 + \pi(\alpha_2)$;

(3) $p(\alpha_1 > \alpha_2) = 0 \Leftrightarrow a_2 \ge a_1 + \pi(\alpha_1)$;

(4) (complementarity) $p(\alpha_1 > \alpha_2) + p(\alpha_2 > \alpha_1) = 1$, especially, if $\alpha_1 = \alpha_2$, then $p(\alpha_1 > \alpha_2) = p(\alpha_2 > \alpha_1) = \frac{1}{2}$;

(5) If $a_1 \le a_2, b_1 \le b_2$, then $p(\alpha_1 > \alpha_2) \ge \frac{1}{2}$ if and only if $a_1 - b_1 \ge a_2 - b_2$; furthermore, $p(\alpha_1 > \alpha_2) = \frac{1}{2}$ if and only if $a_1 - b_1 = a_2 - b_2$;

(6) If $a_1 \ge a_2, b_1 \le b_2$, then $p(\alpha_1 > \alpha_2) \ge 0.5$;

(7) (transitivity) For $\alpha_1 = (a_1, b_1)$, $\alpha_2 = (a_2, b_2)$, $\alpha_3 = (a_3, b_3)$, if $p(\alpha_1 > \alpha_2) > \frac{1}{2}$ and $p(\alpha_2 > \alpha_3) \ge \frac{1}{2}$ or $p(\alpha_1 > \alpha_2) \ge \frac{1}{2}$ and $p(\alpha_2 > \alpha_3) > \frac{1}{2}$, then $p(\alpha_1 > \alpha_3) > \frac{1}{2}$; if $p(\alpha_1 > \alpha_2) = \frac{1}{2}$ and $p(\alpha_2 > \alpha_3) = \frac{1}{2}$, then $p(\alpha_1 > \alpha_3) = \frac{1}{2}$.

For the IFN $\alpha = (a, b)$, Chen and Tan[6] defined the score function $S(\alpha) = a - b$ and used it to compare two IFNs. Next we show the possibility degree can achieve the same ranking as the score function.

Theorem 2.2 For any two IFNs $\alpha_1 = (a_1, b_1)$ and $\alpha_2 = (a_2, b_2)$, $p(\alpha_1 > \alpha_2) \ge \frac{1}{2}$ if and only if $S(\alpha_1) \ge S(\alpha_2)$; furthermore $p(\alpha_1 > \alpha_2) = \frac{1}{2}$ if and only if $S(\alpha_1) = S(\alpha_2)$.

Proof Suppose that $S(\alpha_1) \ge S(\alpha_2)$, then $a_1 - b_1 \ge a_2 - b_2$. Thus $(a_1 + \pi(\alpha_1)) - a_2 \ge (a_2 + \pi(\alpha_2)) - a_1$. If $(a_2 + \pi(\alpha_2)) - a_1 \le 0$, then by (2) in Theorem 2.1, we have $p(\alpha_1 > \alpha_2) = 1$; If $(a_2 + \pi(\alpha_2)) - a_1 > 0$, then $(a_1 + \pi(\alpha_1)) - a_2 > 0$. Therefore

$$\begin{aligned} p(\alpha_1 > \alpha_2) &= \frac{(a_1 + \pi(\alpha_1)) - a_2}{\pi(\alpha_1) + \pi(\alpha_2)} \\ &= \frac{(a_1 + \pi(\alpha_1)) - a_2}{a_1 + \pi(\alpha_1) - a_2 + (a_2 + \pi(\alpha_2) - a_1)} \ge \frac{1}{2}. \end{aligned}$$

If $S(\alpha_1) = S(\alpha_2)$, then $a_1 - b_1 = a_2 - b_2$. Suppose that $a_1 \le a_2$, we have $b_1 \le b_2$. Also by (5) in Theorem 3.1, $p(\alpha_1 > \alpha_2) = \frac{1}{2}$.

On the other hand, let $p(\alpha_1 > \alpha_2) \ge \frac{1}{2}$. Thus $p(\alpha_1 > \alpha_2) \ne 0$. Also from (3) in Theorem 2.1, we have $a_2 < a_1 + \pi(\alpha_1)$.

If $a_1 > a_2 + \pi(\alpha_2)$, then $a_1 - a_2 > a_2 + \pi(\alpha_2) - (a_1 + \pi(\alpha_1)) = b_1 - b_2$. Then $a_1 - b_1 > a_2 - b_2$ and $S(\alpha_1) > S(\alpha_2)$; If $a_1 \le a_2 + \pi(\alpha_2)$, then

$$p(\alpha_1 > \alpha_2) = \frac{a_1 + \pi(\alpha_1) - a_2}{\pi(\alpha_1) + \pi(\alpha_2)}.$$

Also since $p(\alpha_1 > \alpha_2) \ge \frac{1}{2}$, we obtain that

$$\frac{1 - b_1 - a_2}{1 - a_1 - b_1 + 1 - a_2 - b_2} \ge \frac{1}{2}.$$

Therefore, $a_1 - b_1 \ge a_2 - b_2$ and $S(\alpha_1) \ge S(\alpha_2)$. In conclusion, we can prove that if $p(\alpha_1 > \alpha_2) \ge \frac{1}{2}$, then $S(\alpha_1) \ge S(\alpha_2)$.

Let $p(\alpha_1 > \alpha_2) = \frac{1}{2}$. If $\pi(\alpha_1) \ne 0$ and $\pi(\alpha_2) \ne 0$, then $a_2 < a_1 + \pi(\alpha_1)$ and $a_1 < a_2 + \pi(\alpha_2)$. So

$$\begin{aligned} p(\alpha_1 > \alpha_2) &= \frac{a_1 + \pi(\alpha_1) - a_2}{\pi(\alpha_1) + \pi(\alpha_2)} \\ &= \frac{1 - b_1 - a_2}{1 - a_1 - b_1 + 1 - a_2 - b_2} = \frac{1}{2}, \end{aligned}$$

which implies $a_1 - b_1 = a_2 - b_2$ and $S(\alpha_1) = S(\alpha_2)$. Similarly, we can prove if $\pi(\alpha_1) \ne 0, \pi(\alpha_2) = 0$ or $\pi(\alpha_1) = 0$ and $\pi(\alpha_2) \ne 0$, then $S(\alpha_1) = S(\alpha_2)$. If $\pi(\alpha_1) = \pi(\alpha_2) =$

0, then $a_1 = b_1$ and $a_2 = b_2$. Thus $S(\alpha_1) = S(\alpha_2) = 0$. In conclusion, we can prove if $p(\alpha_1 > \alpha_2) = \frac{1}{2}$, then $S(\alpha_1) = S(\alpha_2)$.

By Theorem 2.2, we can conclude that both the proposed possibility degree formula and the score function defined by Chen and Tan[6] bring out the same rank for any two IFNs α_1, α_2. Suppose $\alpha_1 \succ \alpha_2$. The result from our proposed method indicates not only the rank order of α_1, α_2, but also the amount of the possibility, i.e. $p(\alpha_1 > \alpha_2)$ of $\alpha_1 \succ \alpha_2$, which reflects the uncertainty of IFSs.

B. Possibility degree method for ranking n intuitionistic fuzzy numbers

Here we introduce the possibility degree method for ranking n intuitionistic fuzzy numbers $\alpha_1, \alpha_2, \cdots, \alpha_n$.

Step 1 By using pairwise comparisons among n intuitionistic fuzzy numbers $\alpha_1, \alpha_2, \cdots, \alpha_n$, we construct a possibility degree matrix P:

$$P = \begin{pmatrix} 1/2 & p_{12} & \cdots & p_{1n} \\ p_{21} & 1/2 & \cdots & p_{2n} \\ \vdots & \vdots & & \vdots \\ p_{n1} & p_{n2} & \cdots & 1/2 \end{pmatrix}$$

where

$$p_{ij} = p(\alpha_1 > \alpha_2)$$
$$= \frac{\max\{0, (a_1 + \pi(\alpha_1)) - a_2\} - \max\{0, a_1 - (a_2 + \pi(\alpha_2))\}}{\pi(\alpha_1) + \pi(\alpha_2)}.$$

Step 2 Construct the preference relation matrix M from the possivility degree matrix P:

$$M = \begin{pmatrix} 0 & m_{12} & \cdots & m_{1n} \\ m_{21} & 0 & \cdots & m_{2n} \\ \vdots & \vdots & & \vdots \\ m_{n1} & m_{n2} & \cdots & 0 \end{pmatrix}$$

where for any $i \neq j$,

$$m_{ij} = \begin{cases} 1, & p_{ij} \geq 0.5, \\ 0, & p_{ij} < 0.5. \end{cases}$$

Setp 3 Find out the rows in which the elements are all equal to 0 in M. The set of these rows is marked as J. Supposing that $J = \{j_1, j_2, \cdots, j_s\}$, the corresponding compared IFNs $\alpha_{j_1}, \alpha_{j_2}, \cdots \alpha_{j_s}$ are indifferent. Let $X_1 = \{\alpha_{j_1}, \alpha_{j_2}, \cdots \alpha_{j_s}\}$. Remove the elements in rows $j_1, \cdots, j_s$ and columns $j_1, \cdots, j_s$ from the matrix M, and the remained elements construct the matrix M_1. Find out the rows in which the elements are all equal to 0 in M_1, and X_2 denotes the set of corresponding IFNs, which are also indifferent. Continuing the process, we can divide the set of n intuitionistic fuzzy numbers into $X_1, X_2, \cdots, X_l$.

Step 4 If X_i just has one element α_{k_i}, then the rank of IFNs fuzzy numbers $\alpha_1, \alpha_2, \cdots, \alpha_n$ is

$$\alpha_{k_n} \overset{p(\alpha_{k_n} > \alpha_{k_{n-1}})}{\succ} \alpha_{k_{n-1}} \overset{p(\alpha_{k_{n-1}} > \alpha_{k_{n-2}})}{\succ} \cdots \overset{p(\alpha_{k_3} > \alpha_{k_2})}{\succ} \alpha_{k_2} \overset{p(\alpha_{k_2} > \alpha_{k_1})}{\succ} \alpha_{k_1};$$

if there are several IFNs in X_i, we may let $X_i = \{\alpha_k, \alpha_m, \alpha_l\}$, then we obey the following rules to rank $\alpha_k, \alpha_m, \alpha_l$: calculate the average possibility degree which α_i is superior to other intuitionistic fuzzy numbers by the formula

$$w_i = \frac{1}{n} \sum_{j=1}^{n} p_{ij}, \quad i = k, m, l.$$

If $w_k > w_m > w_l$, then $\alpha_k \succ \alpha_m \succ \alpha_l$. And if $w_k = w_m$, then $\alpha_m \sim \alpha_k$.

Remark The above method for ranking n intuitionistic fuzzy numbers is different from Wang's method for ranking interval numbers [10], since we consider the case that there exist some elements that are equal to 0.5 in possibility degree matrix P. Our method is also different from the Xu's method [11] due to different mechanism.

III. A DECISION-MAKING METHOD BASED ON INTUITIONISTIC FUZZY INFORMATION

For a MCDM problem, let $X = \{x_1, x_2, \cdots, x_m\}$ be a set of options, $C = \{c_1, c_2, \cdots, c_n\}$ be a set of criteria and $D = (\alpha_{ij})_{m\times n} = ((a_{ij}, b_{ij}))_{m\times n}$ be a decision making matrix, where the degree of satisfiability and non-satisfiability of each option x_i (i = 1, 2, ...,m) under the criterion c_j is expressed via intuitionistic fuzzy number (a_{ij}, b_{ij}). Let $w = (w_1, w_2, \cdots, w_n)$ be the weight vector of criteria. Decision maker's goal is to obtain the ranking order of the options $x_1, x_2, \cdots, x_m$.

Next we introduce a ranking method based on IFNs, which involves the following steps:

Step I By using weighted average operator or weighted geometric mean operator based on IFNs in [6], we aggregate the elements $\alpha_{ij}(j = 1, 2, 3, \cdots n)$ in i row, and obtain the comprehensive evaluation value α_i of option x_i:

$$\alpha_i = f_w(\alpha_{i1}, \alpha_{i2}, \cdots, \alpha_{in}) = \sum_{j=1}^{n} w_j \alpha_{ij}, \quad i = 1, 2, \cdots, m$$

or

$$\alpha_i = g_w(\alpha_{i1}, \alpha_{i2}, \cdots, \alpha_{in}) = \prod_{j=1}^{n} \alpha_j^{w_j}, \quad i = 1, 2, \cdots, m.$$

Step II Compare $\alpha_1, \alpha_2, \cdots, \alpha_m$ by using the possibility degree method for ranking IFNs.

Example 1 Assume that there are 5 criteria for evaluation of candidates for senior positions: morality(C_1), job attitude(C_2), work style(C_3), knowledge structure (C_4)

and leadership(C_5). The weight vector of criteria is $w = (0.20, 0.10, 0.25, 0.30, 0.15)$.

Suppose that there are 5 candidates $A_1, A_2, \cdots, A_5$, and the information from the assessment under the criteria is respected by IFNs. The corresponding decision making matrix

$$D = \begin{pmatrix} (0.3,0.5) & (0.2,0.6) & (0.6,0.1) & (0.2,0.4) & (0.1,0.8) \\ (0.1,0.7) & (0.1,0.8) & (0.6,0.3) & (0.8,0.1) & (0.2,0.7) \\ (0.4,0.3) & (0.7,0.1) & (0.2,0.6) & (0.2,0.7) & (0.2,0.6) \\ (0.1,0.7) & (0.1,0.8) & (0.1,0.7) & (0.2,0.7) & (0.8,0.1) \\ (0.4,0.5) & (0.7,0.2) & (0.3,0.3) & (0.1,0.7) & (0.1,0.8) \end{pmatrix}$$

Now we rank the candidates $A_1, A_2, \cdots, A_5$ by the above decision making method.

Using weighted average operator for IFNs, we obtain comprehensive evaluation values for 5 candidates:
$\alpha_1 = (0.3330, 0.3417)$, $\alpha_2 = (0.5402, 0.3202)$,
$\alpha_3 = (0.3153, 0.4573)$, $\alpha_4 = (0.3067, 0.5298)$,
$\alpha_5 = (0.3017, 0.4766)$.

By Step 1 and Step 2 in the possibility degree method for ranking IFNs, we obtain the possibility degree matrix P and the preference relation matrix M:

$$P = \begin{pmatrix} 0.5 & 0.254 & 0.6206 & 0.7193 & 0.6519 \\ 0.746 & 0.5 & 0.9932 & 1 & 1 \\ 0.3794 & 0.0068 & 0.5 & 0.6037 & 0.5366 \\ 0.2807 & 0 & 0.3963 & 0.5 & 0.4374 \\ 0.3481 & 0 & 0.4634 & 0.5626 & 0.5 \end{pmatrix},$$

$$M = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

By Step 3 and Step 4 of the possibility degree method, we get the ranking result of 5 candidates:

$$A_2 \overset{0.746}{\succ} A_1 \overset{0.6206}{\succ} A_3 \overset{0.5366}{\succ} A_5 \overset{0.5626}{\succ} A_4.$$

If we adopt the score function $S(\alpha) = a - b$ to rank $\alpha_1, \alpha_2, \cdots, \alpha_5$, we get $S(\alpha_1) = -0.0087$, $S(\alpha_2) = 0.2200$, $S(\alpha_3) = -0.1420$, $S(\alpha_4) = -0.2231$ and $S(\alpha_5) = -0.1749$. Then the ranking of 5 candidates is $A_2 \succ A_1 \succ A_3 \succ A_5 \succ A_4$.

Obviously, the rank of the 5 candidates is same by using our possibility degree method and the score function. While the possibility degree method provides more information to the decision makers, as we may be more certain of A_2 is superior to A_1 than A_3 is superior to A_5.

IV. CONCLUSION

With the easier information acquisition, IFNs are used to represent the degree of satisfiability and non-satisfiability of each alternative with respect to a set of criteria for MCDM problems. In order to rank those alternatives represented by IFNs, we need to adopt reasonable methods to compare IFSs. In this paper, by defining two possibility degree formulas to compare two IFNs, we propose the method of ranking n IFNs and its application to multi-criteria decision making. This method brings the same ranking order of IFNs as that derived by the score function defined by Chen and Tan. Moreover, the proposed possibility degree method provide additional information for the comparison of IFNs. Like the score function defined by Chen and Tan, there is the case that some IFNs are indifferent by using the possibility degree method. For that, we may compare the IFNs by combining the possibility degree method with other methods, such as the accuracy function.

ACKNOWLEDGMENT

This research was supported by the National Basic Research Program of China (973 Program) under Grant No. 2010CB731405 and Shandong Higher Educational Science and Technology Program Project No.J09LA14.

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