A Study of Collective Action Threshold Model Based on Utility and Psychological Theories

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Abstract. In this paper, we extend Granovetter's classic threshold model by adding both utility and psychological threshold. We conduct simulations with the presented model while also considering the spatial factor and friendship influence strength. We observe that the equilibrium of collective dynamics is not closely related to the friendship impact. With no utility and psychological threshold, the equilibrium state of the model is sensitive to the fluctuation of the collective threshold distribution and displays critical phenomena. By comparison, the equilibrium state with considering utility and psychological threshold looks positively robust. Furthermore, we observe that both cases demonstrate group bi-polarization pattern with the increase of standard deviation of the threshold.

Keywords: psychological threshold, utility, collective action, threshold model.

1 Introduction

Models of collective actions are developed for a variety of situations such as innovation[1], migration, and rumor diffusion, riot behavior, strikes and voting [2,4]. López-Pintado and Watts (2008) classified the existing models into two main categories, heuristic and utility models, respectively [7]. The heuristic models, which mainly include Bass's model of diffusion (Bass, 1969) [3] and threshold model of adoption (Granovetter, 1978) [5] correspond to plausible descriptions of how an individual may adopt a new product or practice as a function of the adoptions of others. The collective action can be viewed as the "domino effect" in threshold models, a quite usual phenomenon from our empirical observation for many radical events. For example, a riot ignited from a small group of radical actors might activate "collective consciousness" of a bulk of people. The main advantage of threshold model is concise and feasible; that is, once the activation rule is specified, the equilibrium, or even non-equilibrium of the collective action is relatively straightforward to compute.

Yet lack of general assumptions of the micro mechanisms results in qualitatively different properties for collective action, then studying the principles of collective dynamics from economic or psychological roots for micro details of the process

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seems to be meaningful exploration, such as the interdisciplinary work on group decision making [4].

In economics, utility is a representation of preferences over some set of goods and services, typically expressed by a utility function about the individual preferences. Utility models address the psychological or economic considerations along the decision making process. Later Amos Tversky and Daniel Kahneman developed prospect theory, assigning behavioral implications to value (utility) function to explain irrational human economic choices [6].

The parameters in utility models are interpretable and policy implications are, at least in principle, clear. However, the complicated model design and absence of unified forms inhibit its further development.

In this paper, we present a new model of collective action by taking advantages of both threshold and utility models. Moreover, another two factors, the spatial factor and friendship influence are also considered. The rest of the paper is organized as follows: in Section 2 we brief the classic Granovetter's threshold model mechanism; in Section 3 we provide the new model with utility and psychological threshold. In Section 4 we compare our new model with the classic Granovetter's model through numeric simulation. Section 5 is our conclusion remarks.

2 Granovetter's Threshold Model

Granovetter's threshold model is one of the classic models about collective actions, such as riots and strikes etc. The model assumes that the possibility that one actor would join the collective action depends on the proportion of the actors who have participated in the action. In one social group, each member has one's specific activation threshold, and the threshold of the whole group is subject to certain probability distribution. The threshold for an instigator is zero; for the radical is lower than that for the conservative. The strict mathematic form of threshold model is as shown in Equation (1)

$$F(x) = \int_0^x f(x)dx \qquad (1)$$

where $f(x)$ is the probability distribution of the group threshold x, and $F(x)$ is the corresponding cumulative distribution function, i.e., $F(x)$ stands for the proportion of those actors whose thresholds are equal or less than x. We assume that at the certain time step t, the ratio of the actors who have joined the collective action is $r(t)$, then at step $t + 1$ the proportion of actors who join the action is $r(t + 1) = F(r(t))$. When $r(t + 1) = r(t)$, it is said that collective action reaches the equilibrium state [4].

Next we analyze the mechanism of collective action from economic and psychological aspects and investigate the collective action equilibrium while combing spatial and friendship factors during numeric simulation.

3 The Threshold Model with Both Utility and Psychological Threshold

In the collective action, the decision that each individual joins the collective action depends on the tradeoff between his/her benefit and cost. For example, the reason that a radical instigator has lower activation threshold is that his/her active action could bring more economic or political benefits than others, i.e. participating the action could bring more benefit than cost. For this reason a jobless person might join the strike with a higher possibility than actors with stable living. So the thresholds of all actors are heterogeneous, since the intention, background, benefit and cost of each actor are different. Here we abstract those differences among actors from economic utility point of view, formally for certain collective action. Actor i has benefit b_i and cost c_i, the corresponding utility u_i for i is defined as

$$u_i = b_i - c_i \tag{2}$$

Except the tradeoff between benefit and cost, another factor is local social network information or local social signal. For example, friends have high impact than ordinaries. Provided that actor i has a neighborhood with size N_i, $w_{ij} = 2,1$ stands for the influence strength between friends and ordinary actors respectively. Let $N = \{1, ... n\}$ be a finite but large set of individuals and $a_i \in \{0, 1\}$ be the common set of actions. That is, every individual makes a binary decision (e.g., whether or not to adopt a certain behavior), then we have local social signal or local social pressure defined in [7] as following

$$k_i = \frac{\sum_{j \in N} w_{ij} a_j}{N - 1} \tag{3}$$

Equ.(3) denotes the local information that actor i may access to. Value function of local social signal is named the network effect or network externalities. The externalities arise when the utility assigned to an action explicitly depends on the absolute or relative number of individuals choosing the action. Our analogy is that actor's utility will increase if he/she observes more and more people participate into the action, while the possible punishment or cost will decrease.

In this paper, we specify the value function form as $v(x) = 1 - e^{-\gamma x}, x \geq 0$, where γ denotes the parameter of risk aversion. According to the above analysis, it is straightforward to show that the utility function can be expressed as

$$u_i = b_i - c_i + v(k_i) \tag{4}$$

Based on classic Granovetter's threshold model, we adopt the psychological threshold which is applied to measure the critical point of some psychological stimulation. As one experimental verified concept, the psychological threshold illustrates that human psychological feeling keeps relatively stable until some stimuli reach the critical level. We use psychological threshold to describe actor would not participate the collective action if his/her utility value is less than his/her psychological acceptable critical level. Through this principle we bring economic and psychological impliations into the classic Granovetter's model. Our primary assumption is that collective action is rooted from psychological acceptable basis with utility connotation. It is difficult to quantify the complicated and varying human decision-making process. However, the threshold mechanism illustrates comparatively a simple and effective way, whatever the stimuli are from group pressure, learning, utility motivation or other physiological, psychological or social factors.

Assume that each actor has different psychological tolerable threshold p_i . To measure the individual difference between practical utility and acceptable threshold p_i , we adopt a satisfying level e_i --- a behavioral tendency proposed by March and Simon while satisfying is suboptimal when judged by forward-looking game-theoretic criteria [8]. It may be more effective in leading agents out of social traps than other more sophisticated decision rules [9]. Here we define the satisfying level e_i as the following concise form

$$e_i = u_i - p_i \tag{5}$$

Obviously $e_i \geq 0$ means that the utility value of actor i who joins the action is larger than or equal to the actor i 's corresponding psychological acceptable threshold. If we assume p_i is subject to uniform distribution $m(x)$ with interval $[a, b]$, the expected satisfying level of actor i is defined as

$$E_i = \int_a^b e_i m(x)dx \tag{6}$$

Provided that group utility threshold T_i subject to normal distribution $f(x)$, when the individual expected satisfying level $E_i \geq T_i$, actor i would like to participate the action, conversely i would not join the action.

Next we conduct simulations toward the above model with considering spatial and friendship influence. We suppose group utility is subject to normal distribution (this distribution is more interesting, because it gives a good description of population averages, for example central limit theorem).

4 Simulation and Results Analysis

For simulation we choose N=100 and put actors on periodic lattice with local eight neighbors while friend influence strength equals to 2 and ordinary impact is 1. Three variables, b_i (benefit), c_i (cost) and p_i (threshold) ~ U(0,1) and $\gamma = 1.5$. Fig.1 shows the equilibrium state of collective action by setting friendship density be 0.1 and without considering utility and psychological threshold. In Fig. 1, the x-axis represents group average activation threshold, the y-axis for the standard deviation of group activation threshold, the pixels represent the number of actors who enter into the action when collective action reaches the equilibrium state. This diagram shows that there exists a critical standard deviation for every mean between 2 and 50. The corresponding critical curve is clearly distinguishable and seems to grow more or less linearly with the mean. When the mean is close to 50, the collective action is in unstable state, any small perturbation may lead to some unexpected equilibrium.

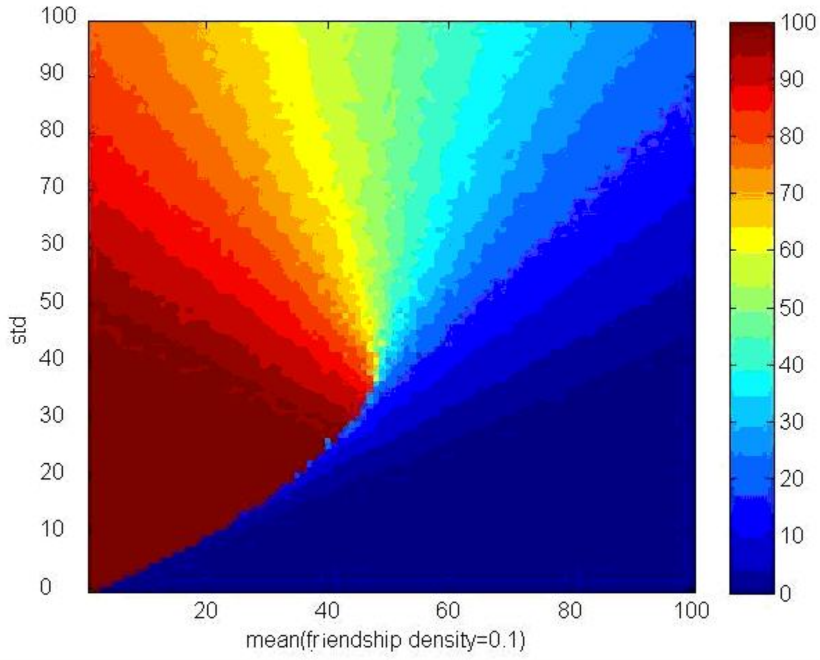

Fig. 1. Collective action equilibrium state vs. activation mean and standard deviation, without considering utility and psychological threshold, and with friendship density= 0.1

From Fig.2, we observe other interesting critical points around 12 and 33 of activation standard deviation. This equilibrium state remains unchanged until the standard deviation is close to 33. We also find that the friendship density or friend influence strength seem have no evident impact on the final equilibrium of collective action (in Fig.2 the density of friendship varies from 0 to 1; different color represents different friend density).

For comparison, we conduct simulation by considering utility and psychological threshold, with results as shown in Fig.3. We fix friend density 0.1 since it does not affect final equilibrium results.

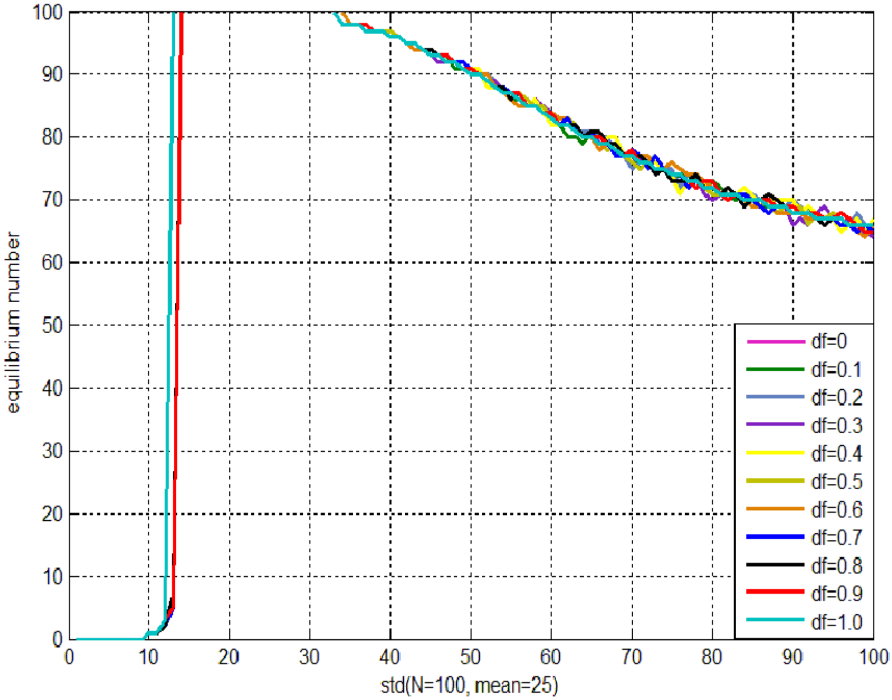

Fig. 2. Collective action equilibrium state vs. activation standard deviation and friendship density (activation mean is fixed at 25)

Fig.3 shows that within wide range of activation mean (from 1 to 50), the fluctuation of activation standard deviation results in unstable collective equilibrium. We also observe a vague critical line between standard deviation and mean, which is not as sharp as shown in Fig.1. The critical phenomena do not appear at activation mean equal to 50 as illustrated in Fig.1, either.

Furthermore, in order to investigate the equilibrium dynamics of collective action with and without utility and psychological threshold implication, we undertake simulations for each case[1]. The simulation on the threshold model of collective action without adopting utility and psychological implication demonstrates that group unstable critical phenomenon appears when the average activation threshold is less than 0.5 (in proportion) and standard deviation is close to 0.25 (in proportion) as shown in Fig.4. When the activation mean is 0.5 and the standard deviation is 0.25 the equilibrium number of collective action suddenly jumps from 12 to 100, i.e. all actors join the action.

[1] Since friends density does not significantly affect the collective action equilibrium, for the simulations in Fig.4 and Fig.5 we fix density at 0.1.

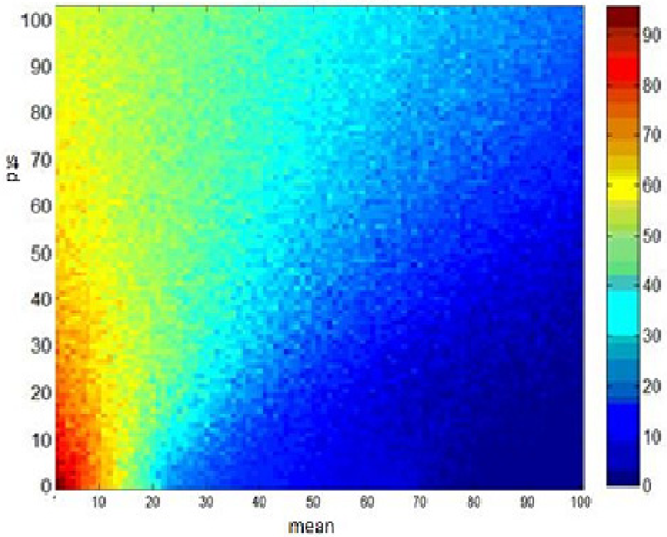

Fig. 3. Collective action equilibrium states under different activation mean and standard deviation when considering utility and psychological thresholds (friendship density 0.1)

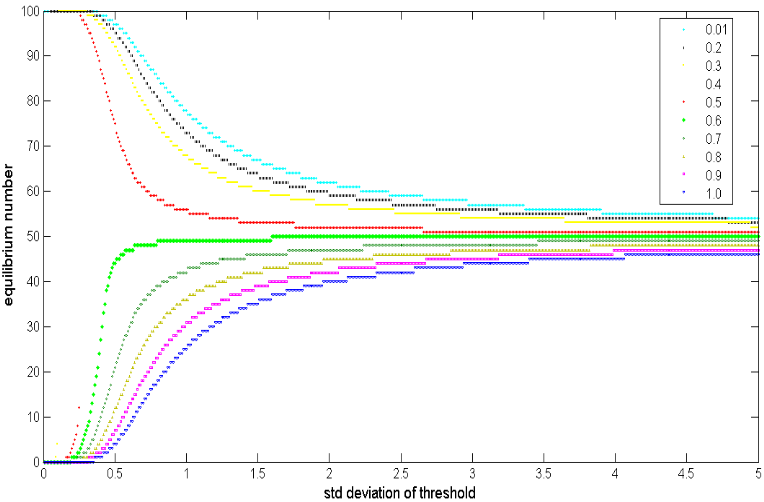

Fig. 4. Given average activation threshold, the impact on equilibrium state as a result of the variation of threshold standard deviation without considering utility and psychological thresholds

In distinct contrast to the above results, the simulation of the model with the involvement of utility implication and psychological threshold indicates that collective action equilibrium reveals stable transitional state, i.e. no critical phenomenon is found as shown in Fig.5.

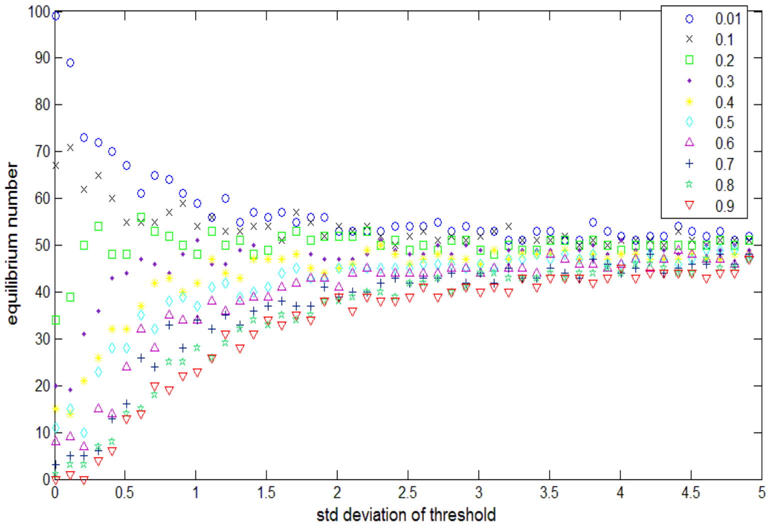

Fig. 5. Given average activation threshold, the impact on equilibrium state as a result of the variation of threshold standard deviation by considering utility and psychological thresholds

However the unexpected result is that both cases show common trend, the collective action displays bi-polarization pattern (the ratio between number of join and not join is approximate to 50% to 50%) with the increase of standard deviation at a variety of the activation mean. As respect to the collective stability of the equilibrium sate and the possibility of phase transition, Galam (2005) presented a general sequential probabilistic frame which shows how various different micro rules lead to the same either ordered or disordered phase [10].

5 Conclusion

In this paper, we present a new model by adopting both utility and psychological thresholds on the basis of Granovetter's threshold model. We investigate the collective action equilibrium through numeric simulations with consideration of spatial and friendship influence strength. We find that the equilibrium is not closely related to friendship impact. With no considering utility and psychological factors, the equilibrium of the model is sensitive to the variation of group threshold distribution and displays critical phenomena. On the contrary, when considering utility and psychological factors, the collective equilibrium is more robust to the threshold distribution.

Furthermore, we observe that both cases demonstrate group bi-polarization pattern with the increase of the standard deviation. Our preliminary conclusion is that the classic threshold model is more appropriate for describing riot, strike and similar unexpected outbreak of social events and the model of utility and psychological threshold is suitable to depict human economic behaviors such as technological spreading.

Granovetter's classic threshold model and other literatures about collective models may lack clear economic, management or psychological implications. We suppose that the collective action models desperately need more interdisciplinary contribution to analyze or explain human collective behaviors such as that from economics, sociology, social psychology, anthropology etc. More experimental and practical evidences are also needed to verify the effectiveness of the proposed model.

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