Intuitionistic Fuzzy Dependent OWA Operator and Its Application

Cuiping Wei[1,2] , Xijin Tang[3] and Yanzhao Bi[1]

[1] College of Mathematical Sciences, Yangzhou University, Yangzhou 225002, China
[2] Management School, Qufu Normal University, Rizhao 276826, China
[3] Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China

Abstract. In this paper, we propose a novel approach, based on entropy and similarity measure of intuitionistic fuzzy sets, to determine weights of the IFOWA operator. Then we define a new intuitionistic fuzzy dependent OWA (IFDOWA) operator which is applied to handling multi-attribute group decision making problem with intuitionistic fuzzy information. Finally, an example is given to demonstrate the rationality and validity of the proposed approach.
Keywords: Multi-attribute group decision making, intuitionistic fuzzy values, intuitionistic fuzzy dependent OWA operator, entropy, similarity.

1 Introduction

The ordered weighted aggregating (OWA) operator [26], as an important tool for aggregating information, has been investigated and applied in many documents [1, 11, 20, 27]. One critical issue of the OWA operator is to determine its associated weights. Up to now, a lot of methods have been proposed to determine the OWA weights. Xu [19] classified all those weight-determining approaches into two categories: argument-independent approaches [6, 11, 14, 20, 23, 26] and argument-dependent approaches [1, 7, 19, 21, 24, 25]. For the 1st category, Yager [26] suggested an approach to compute the OWA weights based on linguistic quantifiers provided by Zadeh [28, 29]. O'Hagan [11] defined degree of orness and constructed a nonlinear programming to obtain the weights of OWA operator. Xu [20] made an overview of methods for obtaining OWA weights and developed a novel weight-determining method using the idea of normal distribution. For the 2nd category, Filev and Yager [7] developed two procedures to determine the weights of OWA operator. Xu and Da [21] established a linear objective-programming model to obtain the OWA weights. Xu [19] proposed a new dependent OWA operator which can relieve the influence of unfair arguments on the aggregated results. In [24, 25], Yager and Filev developed an argument-dependent method to generate the OWA weights with power function of the input arguments.

Andrzej M.J. Skulimowski (ed.): Proceedings of KICSS'2013, pp. 298-309

With the growing research of intuitionistic fuzzy set theory [2, 3] and the expansion of its application, it is more and more important to aggregate intuitionistic fuzzy information effectively. Xu [18, 22] proposed some intuitionistic fuzzy aggregation operators to aggregate the intuitionistic fuzzy information. In [18], Xu pointed out that the the intuitionistic fuzzy OWA (IFOWA) weights can be obtained similar to the OWA weights, such as the normal distribution based method. However, the characteristics of the input arguments are not considered in these methods.

In this paper, we investigate the IFOWA operator, and establish a new argument-dependent method to determine the IFOWA weights. To do that, this paper is organized as follows. Section 2 reviews the basic concepts about intuitionistic fuzzy information. In Section 3, a new argument-dependent approach to obtain the IFOWA weights is proposed based on entropy and similarity measure. A intuitionistic fuzzy dependent OWA (IFDOWA) operator is developed and its properties are studied. Section 4 provides a practical approach to solve multi-attribute group decision making problem with intuitionistic fuzzy information based on IFDOWA operator. The concluding remarks are given in Section 5.

2 Preliminaries

Some basic concepts of intuitionistic fuzzy sets, some operators, entropy and similarity measures are reviewed.

2.1 The OWA operator and intuitionistic fuzzy sets

Definition 2.1 [26] Let $(a_1, a_2, \cdots, a_n)$ be a collection of numbers. An ordered weighted averaging (OWA) operator is a mapping: $R^n \to R$, such that

$$\mathrm{OWA}(a_1, a_2, \cdots, a_n) = w_1 a_{\sigma(1)} + w_2 a_{\sigma(2)} + \cdots + w_n a_{\sigma(n)}, \tag{1}$$

where $a_{\sigma(j)}$ is the jth largest of $a_j (j = 1, 2, \cdots, n)$, and $w = (w_1, w_2, \cdots, w_n)^T$ is an associated vector of the operator with $w_j \in [0, 1]$ and $\sum\limits_{j=1}^{n} w_j = 1$.

Definition 2.2 [2, 3] Let X be a universe of discourse. An intuitionistic fuzzy set (IFS) in X is an object with the form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{2}$$

where $\mu_A : X \to [0, 1]$, $\nu_A : X \to [0, 1]$ with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1, \forall x \in X$. The numbers $\mu_A(x)$ and $\nu_A(x)$ denote the degree of membership and non-membership of x to A, respectively.

For each IFS A in X, we call $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ the intuitionistic index of x in A, which denotes the hesitancy degree of x to A.

For convenience, we call $\alpha = (\mu_\alpha, \nu_\alpha)$ an intuitionistic fuzzy value (IFV) [22], where $\mu_\alpha \in [0,1], \nu_\alpha \in [0,1]$, and $\mu_\alpha + \nu_\alpha \leq 1$. Let Θ be the universal set of IFVs.

For comparison of IFVs, Chen and Tan [5] defined a score function while Hong and Choi [8] defined an accuracy function. Based on the two functions, Xu [22] provided a method to compare two intuitionistic fuzzy values (IFVs).

Definition 2.3 [22] Let $\alpha = (\mu_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \nu_\beta)$ be two IFVs, $s(\alpha) = \mu_\alpha - \nu_\alpha$ and $s(\beta) = \mu_\beta - \nu_\beta$ be the score degrees of α and β, respectively; $h(\alpha) = \mu_\alpha + \nu_\alpha$ and $h(\beta) = \mu_\beta + \nu_\beta$ be the accuracy degrees of α and β, respectively. Then
(1) If $s(\alpha) < s(\beta)$, then α is smaller than β, denoted by $\alpha < \beta$;
(2) If $s(\alpha) = s(\beta)$, then

1)If $h(\alpha) = h(\beta)$, then α and β represent the same information, i.e., $\mu_\alpha = \mu_\beta$ and $\nu_\alpha = \nu_\beta$, denoted by $\alpha = \beta$;

2) If $h(\alpha) < h(\beta)$, then α is smaller than β, denoted by $\alpha < \beta$;

3) If $h(\alpha) > h(\beta)$, then α is bigger than β, denoted by $\alpha > \beta$.

Definition 2.4 [18, 22] Let $\alpha = (\mu_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \nu_\beta)$ be two IFVs. Then two operational laws of IFVs are given as follows:
(1) $\overline{\alpha} = (\nu_\alpha, \mu_\alpha)$;
(2) $\alpha \oplus \beta = (\mu_\alpha + \mu_\beta - \mu_\alpha\mu_\beta, \nu_\alpha\nu_\beta)$;
(3) $\lambda\alpha = (1 - (1 - \mu_\alpha)^\lambda, \nu_\alpha^\lambda), \lambda \geq 0$;
(4) $\lambda(\alpha_1 + \alpha_2) = \lambda\alpha_1 + \lambda\alpha_2$;
(5) $\lambda_1\alpha + \lambda_2\alpha = (\lambda_1 + \lambda_2)\alpha$.

With the thorough research of intuitionistic fuzzy set theory and the continuous expansion of its application scope, it is more and more important to aggregate intuitionistic fuzzy information effectively. Xu [22, 18] proposed some intuitionistic fuzzy aggregation operators to aggregate the intuitionistic fuzzy information.

Definition 2.5 [18] Let $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})(i = 1, 2, \cdots, n)$ be a collection of IFVs. An intuitionistic fuzzy weighted averaging (IFWA) operator is a mapping: $\Theta^n \to \Theta$, such that

$$\text{IFWA}(\alpha_1, \alpha_2, \cdots, \alpha_n) = w_1\alpha_1 \oplus w_2\alpha_2 \oplus \cdots \oplus w_n\alpha_n = \left(1 - \prod_{j=1}^{n} (1 - \mu_{\alpha_j})^{w_j}, \prod_{j=1}^{n} \nu_{\alpha_j}^{w_j}\right) \tag{3}$$

where $w = (w_1, w_2, \cdots, w_n)^T$ is the weighting vector of $\alpha_i (i = 1, 2, \cdots, n)$ with $w_j \in [0, 1]$ and $\sum\limits_{j=1}^{n} w_j = 1$.

Definition 2.6 [18] Let $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})(i = 1, 2, \cdots, n)$ be a collection of IFVs. An intuitionistic fuzzy ordered weighted averaging (IFOWA) operator is a mapping: $\Theta^n \to \Theta$, such that

$$\text{IFOWA}(\alpha_1, \alpha_2, \cdots, \alpha_n) = w_1\alpha_{\sigma(1)} \oplus w_2\alpha_{\sigma(2)} \oplus \cdots \oplus w_n\alpha_{\sigma(n)}$$

$$= \left(1 - \prod_{j=1}^{n}(1-\mu_{\alpha_{\sigma(j)}})^{w_j}, \prod_{j=1}^{n}\nu_{\alpha_{\sigma(j)}}^{w_j}\right) \tag{4}$$

where $\alpha_{\sigma(j)}$ is the jth largest of $\alpha_j (j = 1, 2, \cdots, n)$, and $w = (w_1, w_2, \cdots, w_n)^T$ is an associated vector of the operator with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

2.2 Entropy and similarity measure for IFSs

Introduced by Burillo and Bustince [4], Intuitionistic fuzzy entropy is used to estimate the uncertainty of an IFS. Szmidt and Kacprzyk [12] defined an entropy measure E_{SK} for an IFS. Wang and Lei [13] gave an entropy measure E_{WL}:

$$E_{SK}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{\max Count(A_i \cap A_i^C)}{\max Count(A_i \cup A_i^C)}, \tag{5}$$

where $A_i = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle\}$ is a single element IFS,
$A_i \cap A_i^C = \{\langle x_i, \min\{\mu_A(x_i), \nu_A(x_i)\}, \max\{\mu_A(x_i), \nu_A(x_i)\}\rangle\}$,
$A_i \cup A_i^C = \{\langle x_i, \max\{\mu_A(x_i), \nu_A(x_i)\}, \min\{\nu_A(x_i), \mu_A(x_i)\}\rangle\}$. For every IFS A, $\max Count(A) = \sum_{i=1}^{n}(\mu_A(x_i) + \pi_A(x_i))$ is the biggest cardinality of A.

$$E_{WL}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{\min\{\mu_A(x_i), \nu_A(x_i)\} + \pi_A(x_i)}{\max\{\mu_A(x_i), \nu_A(x_i)\} + \pi_A(x_i)}. \tag{6}$$

Wei and Wang [16] proved that E_{SK} and E_{WL} are equivalent. For convenience, we use the entropy measure E_{WL} in the following.

Based on E_{WL}, the entropy measure for an intuitionistic fuzzy value $\alpha = (\mu_\alpha, \nu_\alpha)$ can be given as:

$$E(\alpha) = \frac{\min\{\mu_\alpha, \nu_\alpha\} + \pi_\alpha}{\max\{\mu_\alpha, \nu_\alpha\} + \pi_\alpha}. \tag{7}$$

Similarity measure [9], another important topic in the theory of intuitionistic fuzzy sets, is to describe the similar degree between two IFSs. Wei and Tang [15] constructed a new similarity measure S_{WT} for IFSs based on entropy measure E_{WL}.

$$S_{WT}(A, B) = \frac{1}{n}\sum_{i=1}^{n}\frac{1 - \min\{|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\}}{1 + \max\{|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\}}. \tag{8}$$

Now we give a similarity measure between two IFVs $\alpha = (\mu_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \nu_\beta)$ based on S_{WT}:

$$S(\alpha, \beta) = \frac{1 - \min\{|\mu_\alpha - \mu_\beta|, |\nu_\alpha - \nu_\beta|\}}{1 + \max\{|\mu_\alpha - \mu_\beta|, |\nu_\alpha - \nu_\beta|\}}. \tag{9}$$

3 IFDOWA operator and its properties

In [18], Xu pointed out that the IFOWA weights can be determined similarly to the OWA weights. For example, we can use the normal distribution based method. However, those methods belong to the category of argument-independent approaches. Here we develop an argument-dependent approach to determine the IFOWA weights based on intuitionistic fuzzy entropy and similarity measure.

We suppose $\alpha_j = (\mu_{\alpha_j}, \nu_{\alpha_j})(j = 1, 2, \cdots, n)$ is a collection of IFVs, $(\alpha_{\sigma(1)}, \alpha_{\sigma(2)}, \cdots, \alpha_{\sigma(n)})$ is a permutation of $(\alpha_1, \alpha_2, \cdots, \alpha_n)$ such that $\alpha_{\sigma(i)} \geq \alpha_{\sigma(j)}$ for all $i \leq j$. The weighting vector of IFOWA operator $w = (w_1, w_2, \cdots, w_n)^T$ is to be determined, such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

During the information aggregating process, we usually expect that the uncertainty degrees of arguments are as small as possible. Thus, the smaller uncertainty degree of argument $\alpha_{\sigma(j)}$, the bigger the weight w_j. Conversely, the bigger uncertainty degree of argument $\alpha_{\sigma(j)}$, the smaller the weight w_j. The uncertainty degrees of arguments can be measured by Formula (7). Thus, the weighting vector of the IFOWA operator can be defined as:

$$w_j^a = \frac{1 - E(\alpha_{\sigma(j)})}{\sum_{j=1}^{n} [1 - E(\alpha_{\sigma(j)})]}, \quad j = 1, 2, \cdots, n. \tag{10}$$

In the following, we define the weighting vector of the IFOWA operator from another viewpoint. In real-life situation, the arguments $\alpha_{\sigma(j)}(j = 1, 2, \cdots, n)$ usually take the form of a collection of n preference values provided by n different individuals. Some individuals may assign unduly high or unduly low preference values to their preferred or repugnant objects. In such a case, we shall assign very small weights to these “false” or “biased” opinions, that is to say, the more similar an argument $\alpha_{\sigma(j)}$ is to others, the bigger the weight w_j. Conversely, the less similar an argument $\alpha_{\sigma(j)}$ is to others, the smaller the weights w_j. The similar degree between two arguments can be calculated by Formula (9).

Definition 3.1 Let $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})(i = 1, 2, \cdots, n)$ be a collection of IFVs, $(\alpha_{\sigma(1)}, \alpha_{\sigma(2)}, \cdots, \alpha_{\sigma(n)})$ is a permutation of $(\alpha_1, \alpha_2, \cdots, \alpha_n)$ such that $\alpha_{\sigma(i)} \geq \alpha_{\sigma(j)}$ for all $i \leq j$. Then the overall similarity degree between $\alpha_{\sigma(j)}$ and other arguments $\alpha_{\sigma(l)}(l = 1, 2, \cdots, n, l \neq j)$is defined as:

$$S(\alpha_{\sigma(j)}) = \sum_{\substack{l=1 \\ l \neq j}}^{n} S(\alpha_{\sigma(j)}, \alpha_{\sigma(l)}), \quad j = 1, 2, \cdots, n. \tag{11}$$

So we define the weighting vector $w = (w_1, w_2, \cdots, w_n)^T$ of the IFOWA operator as following:

$$w_j^b = \frac{S(\alpha_{\sigma(j)})}{\sum\limits_{j=1}^{n} S(\alpha_{\sigma(j)})}, \quad j = 1, 2, \cdots, n. \tag{12}$$

According to the above analysis, the weighting vector of the IFOWA operator associates not only with w^a, but also with w^b. Thus, we use the linear weighting method to derive the combined weighting vector of the IFOWA operator:

$$w_j = \lambda w_j^a + (1-\lambda) w_j^b, \quad \text{where } \lambda \in [0,1], \quad j = 1, 2, \cdots, n. \tag{13}$$

Since $\sum\limits_{j=1}^{n}[1 - E(\alpha_{\sigma(j)})] = \sum\limits_{j=1}^{n}[1 - E(\alpha_j)]$ and $\sum\limits_{j=1}^{n} S(\alpha_{\sigma(j)}) = \sum\limits_{j=1}^{n} S(\alpha_j)$, Formula (10), (12) and (13) can be rewritten as:

$$w_j^a = \frac{1 - E(\alpha_{\sigma(j)})}{\sum\limits_{j=1}^{n}[1 - E(\alpha_j)]}, \quad j = 1, 2, \cdots, n. \tag{14}$$

$$w_j^b = \frac{S(\alpha_{\sigma(j)})}{\sum\limits_{j=1}^{n} S(\alpha_j)}, \quad j = 1, 2, \cdots, n. \tag{15}$$

$$w_j = \frac{\lambda[1 - E(\alpha_{\sigma(j)})]}{\sum\limits_{j=1}^{n}[1 - E(\alpha_j)]} + \frac{(1-\lambda) S(\alpha_{\sigma(j)})}{\sum\limits_{j=1}^{n} S(\alpha_j)}, \tag{16}$$

where $\lambda \in [0,1] \quad j = 1, 2, \cdots, n$.

Definition 3.2 Let $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})(i = 1, 2, \cdots, n)$ be a collection of IFVs. An intuitionistic fuzzy dependent OWA (IFDOWA) operator is a mapping: $\Theta^n \to \Theta$, such that

$$\begin{aligned} \text{IFDOWA}(\alpha_1, \alpha_2, \cdots, \alpha_n) &= w_1 \alpha_{\sigma(1)} \oplus w_2 \alpha_{\sigma(2)} \oplus \cdots \oplus w_n \alpha_{\sigma(n)} \\ &= \left(1 - \prod_{j=1}^{n} (1 - \mu_{\alpha_{\sigma(j)}})^{w_j}, \prod_{j=1}^{n} \nu_{\alpha_{\sigma(j)}}^{w_j}\right) \end{aligned} \tag{17}$$

where $(\alpha_{\sigma(1)}, \alpha_{\sigma(2)}, \cdots, \alpha_{\sigma(n)})$ is a permutation of $(\alpha_1, \alpha_2, \cdots, \alpha_n)$ such that $\alpha_{\sigma(i)} \geq \alpha_{\sigma(j)}$ for all $i \leq j$, $w = (w_1, w_2, \cdots, w_n)^T$ is the associated weighting vector which can be calculated by Formula (16).

By Formula (16) and (17), we obtain

$$\text{IFDOWA}(\alpha_1, \alpha_2, \cdots, \alpha_n) = \mathop{\oplus}\limits_{j=1}^{n} \alpha_{\sigma(j)} \left\{ \frac{\lambda[1 - E(\alpha_{\sigma(j)})]}{\sum\limits_{j=1}^{n}[1 - E(\alpha_j)]} + \frac{(1-\lambda) S(\alpha_{\sigma(j)})}{\sum\limits_{j=1}^{n} S(\alpha_j)} \right\}$$

$$= \overset{n}{\underset{j=1}{\oplus}} \alpha_j \left\{ \frac{\lambda[1-E(\alpha_j)]}{\sum\limits_{j=1}^{n}[1-E(\alpha_j)]} + \frac{(1-\lambda)S(\alpha_j)}{\sum\limits_{j=1}^{n}S(\alpha_j)} \right\} \tag{18}$$

Yager [24] pointed that an OWA operator is called neat if the aggregated value is independent of the ordering. Therefore, the IFDOWA operator is a neat operator. By Formula (16) and (17), we can get the following properties.

Theorem 3.1 Let $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})(i = 1, 2, \cdots, n)$ be a collection of IFVs, $(\alpha_{\sigma(1)}, \alpha_{\sigma(2)},$
$\cdots, \alpha_{\sigma(n)})$ be a permutation of $(\alpha_1, \alpha_2, \cdots, \alpha_n)$ such that $\alpha_{\sigma(i)} \geq \alpha_{\sigma(j)}$ for all $i \leq j$. Suppose $E(\alpha_\sigma(j))$ is the entropy of $\alpha_\sigma(j)$ and $S(\alpha_\sigma(j))$ is the similarity degree between $\alpha_\sigma(j)$ and other arguments. If $E(\alpha_\sigma(i)) \leq E(\alpha_\sigma(j))$ and $S(\alpha_\sigma(i)) \geq S(\alpha_\sigma(j))$, then $w_i \geq w_j$.

Theorem 3.2 Let $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})(i = 1, 2, \cdots, n)$ be a collection of IFVs. If $\alpha_i = \alpha_j$, for all i, j, then $w_j = \frac{1}{n}$ for all j.

Yager [26] further introduced two characterizing measures called dispersion measure and orness measure, respectively, associated with the weighting vector w of the OWA operator, where the dispersion measure of the aggregation is defined as

$$disp(w) = -\sum_{j=1}^{n} w_j \ln w_j, \tag{19}$$

which measures the degree to which w takes into account the information in the arguments during the aggregation. Particularly, if $w_j = 0$ for any j, $disp(w) = 0$; if $w = (\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n})^T$, $disp(w) = \ln n$.

The second one, the orness measure of the aggregation, is defined as

$$orness(w) = \frac{1}{n-1}\sum_{j=1}^{n}(n-j)w_j, \tag{20}$$

which lies in the unit interval $[0, 1]$ and characterizes the degree to which the aggregation is like an *or* operation. Particularly, if $w = (1, 0, \cdots, 0)^T$, $orness(w) = 1$; if $w = (\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n})$, $disp(w) = 0.5$; if $w = (0, \cdots, 0, 1)^T$, $orness(w) = 0$.

From Formulas (16), (19) and (20), it follows that

$$disp(w) = -\sum_{j=1}^{n} \left\{ \frac{\lambda[1-E(\alpha_{\sigma(j)})]}{\sum\limits_{j=1}^{n}[1-E(\alpha_j)]} + \frac{(1-\lambda)S(\alpha_{\sigma(j)})}{\sum\limits_{j=1}^{n}S(\alpha_j)} \right\} \cdot \tag{21}$$

$$\ln \left\{ \frac{\lambda[1-E(\alpha_{\sigma(j)})]}{\sum\limits_{j=1}^{n}[1-E(\alpha_j)]} + \frac{(1-\lambda)S(\alpha_{\sigma(j)})}{\sum\limits_{j=1}^{n}S(\alpha_j)} \right\}.$$

$$orness(w)=\frac{1}{n-1}\sum_{j=1}^{n}(n-j)\cdot\left\{\frac{\lambda[1-E(\alpha_{\sigma(j)})]}{\sum\limits_{j=1}^{n}[1-E(\alpha_j)]}+\frac{(1-\lambda)S(\alpha_{\sigma(j)})}{\sum\limits_{j=1}^{n}S(\alpha_j)}\right\}. \quad (22)$$

Example 3.1 Let $\alpha_1=(0.2,0.5)$, $\alpha_2=(0.4,0.2)$, $\alpha_3=(0.5,0.4)$, $\alpha_4=(0.3,0.5)$, $\alpha_5=(0.7,0.1)$ be a collection of IFVs. The re-ordered argument $\alpha_j(j=1,2,3,4,5)$ in descending order are $\alpha_{\sigma(1)}=(0.7,0.1)$, $\alpha_{\sigma(2)}=(0.4,0.2)$, $\alpha_{\sigma(3)}=(0.5,0.4)$, $\alpha_{\sigma(4)}=(0.3,0.5)$, $\alpha_{\sigma(5)}=(0.2,0.5)$. Suppose $\lambda=0.5$, by Formula (14), (15) and (16), we obtain $w^a=(0.3823,0.1433,0.0956,0.1638,0.2150)$, $w^b=(0.1632,0.2101,0.2145,0.2123,0.1999)$. Thus $w=(0.27275,0.17670,0.15505,0.18805,0.20745)$.

By Formulas (19) and (20), we have

$$disp(w)=-\sum_{j=1}^{5}w_j\ln w_j=1.5902.$$

$$orness(w)=\frac{1}{5-1}\sum_{j=1}^{5}(5-j)w_j=0.3609.$$

By Formulas (17) and (18), we have IFDOWA$(\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5)=(0.4724,0.2648)$. Therefore, the collective argument is $(0.4724,0.2648)$.

4 The application of IFDOWA operator in multi-attribute group decision

In this section, we apply the IFDOWA operator to multi-attribute group decision making problem which can be described as follows.

We suppose $X=\{x_1,x_2,\cdots,x_n\}$ is a set of evaluation alternatives, $D=\{d_1,d_2,\cdots,d_s\}$ is a set of decision makers, $U=\{u_1,u_2,\cdots,u_m\}$ is an attribute set, and $v=(v_1,v_2,\cdots,v_m)^T$ is a weighting vector of attributes such that $v_j\in[0,1]$ and $\sum\limits_{j=1}^{m}v_j=1$. Let $R^{(k)}=\left(r_{ij}^{(k)}\right)_{n\times m}(k=1,2,\cdots,s)$ be intuitionistic fuzzy decision matrices, where $r_{ij}^{(k)}=(\mu_{ij}^{(k)},\nu_{ij}^{(k)})$ is an IFV and provided by the decision maker $d_k\in D$ for the alternative $x_i\in X$ with respect to the attribute $u_j\in U$.

Based on the IFWA operator and the IFDOWA operator, we rank the alternatives $x_i(i=1,2,\cdots,n)$ by the following steps:

Step 1. Utilize the IFWA operator to derive the individual overall aggregated values $z_i^{(k)}(i=1,2,\cdots,n,\ k=1,2,\cdots,s)$ of the alternatives $x_i(i=1,2,\cdots,n)$ by decision makers $d_k(k=1,2,\cdots,s)$, where

$$z_i^{(k)}=\text{IFWA}_v(r_{i1}^{(k)},r_{i2}^{(k)},\cdots,r_{im}^{(k)})=v_1r_{i1}^{(k)}\oplus v_2r_{i2}^{(k)}\oplus\cdots\oplus v_mr_{im}^{(k)}, \quad (23)$$

where $v=(v_1,v_2,\cdots,v_m)^T$ is the weighting vector of the attributes of $u_j(j=1,2,\cdots,m)$, with $v_j\in[0,1]$ and $\sum\limits_{j=1}^{m}v_j=1$.

Step 2. Utilize the IFDOWA operator to derive the overall aggregated values $z_i(i=1,2,\cdots,n)$ of the alternatives $x_i(i=1,2,\cdots,n)$, where

$$z_i=\text{IFDOWA}_w(z_i^{(1)},z_i^{(1)},z_i^{(2)},\cdots,z_i^{(s)})=w_1^{(i)}z_i^{\sigma(1)}\oplus w_2^{(i)}z_i^{\sigma(2)}\oplus\cdots\oplus w_s^{(i)}z_i^{\sigma(s)}, \tag{24}$$

where $w^{(i)}=(w_1^{(i)},w_2^{(i)},\cdots,w_s^{(i)})(i=1,2,\cdots,n)$ are calculated by Formula (16).

Step 3. Utilize the Definition 2.2 to compare the overall aggregated values $z_i(i=1,2,\cdots,n)$ and rank the alternatives $x_i(i=1,2,\cdots,n)$.

We adopt the example used in [10] and [17] to illustrate the proposed approach.

Example 4.1 The information management steering committee of Midwest American Manufacturing Corp. must prioritize for development and implementation a set of six information technology improvement projects $x_i(i=1,2,\cdots,6)$, which have been proposed by area managers. The committee is concerned that the projects are prioritized from highest to lowest potential contribution to the firm's strategic goal of gaining competitive advantages in the industry. In assessing the potential contribution of each project, three factors are considered, u_1: productivity, u_2: differentiation, and u_3: management, whose weight vector is $v=(0.35,0.35,0.30)$. Suppose that there are four decision makers $d_k(k=1,2,3,4)$. They provided their preferences with IFVs $r_{ij}^{(k)}=(\mu_{ij}^{(k)},\nu_{ij}^{(k)})(i=1,2,\cdots,6,j=1,2,3)$ over the projects $x_i(i=1,2,\cdots,6)$ with respect to the factors $u_j(j=1,2,3)$, which are listed as follows:

$$R^{(1)}=\begin{pmatrix}(0.3,0.2)&(0.6,0.1)&(0.5,0.2)\\(0.5,0.1)&(0.3,0.2)&(0.4,0.2)\\(0.4,0.3)&(0.5,0.2)&(0.3,0.1)\\(0.3,0.1)&(0.5,0.3)&(0.3,0.2)\\(0.4,0.3)&(0.5,0.3)&(0.4,0.2)\\(0.5,0.4)&(0.2,0.1)&(0.3,0.2)\end{pmatrix}\quad R^{(2)}=\begin{pmatrix}(0.5,0.3)&(0.2,0.1)&(0.3,0.3)\\(0.3,0.1)&(0.5,0.3)&(0.4,0.2)\\(0.3,0.4)&(0.4,0.3)&(0.3,0.1)\\(0.5,0.3)&(0.6,0.3)&(0.5,0.2)\\(0.5,0.3)&(0.3,0.2)&(0.3,0.2)\\(0.5,0.3)&(0.4,0.3)&(0.2,0.1)\end{pmatrix}$$

$$R^{(3)}=\begin{pmatrix}(0.4,0.2)&(0.5,0.1)&(0.5,0.3)\\(0.4,0.1)&(0.6,0.3)&(0.5,0.2)\\(0.2,0.2)&(0.3,0.1)&(0.5,0.3)\\(0.5,0.4)&(0.6,0.2)&(0.3,0.1)\\(0.6,0.3)&(0.5,0.2)&(0.6,0.2)\\(0.4,0.2)&(0.3,0.1)&(0.5,0.1)\end{pmatrix}\quad R^{(4)}=\begin{pmatrix}(0.3,0.1)&(0.5,0.4)&(0.4,0.3)\\(0.5,0.2)&(0.4,0.3)&(0.7,0.1)\\(0.6,0.1)&(0.4,0.2)&(0.2,0.1)\\(0.3,0.2)&(0.5,0.3)&(0.3,0.2)\\(0.4,0.3)&(0.3,0.1)&(0.2,0.2)\\(0.3,0.1)&(0.5,0.2)&(0.4,0.3)\end{pmatrix}$$

Step 1. Utilize the IFWA operator to derive the individual overall aggregated values $z_i^{(k)}(i=1,2,\cdots,6,\ k=1,2,3,4)$ of the alternatives $x_i(i=1,2,\cdots,6)$ by decision makers $d_k(k=1,2,3,4)$:

$$z_1^{(1)}=(0.4798,0.1569),\ z_2^{(1)}=(0.4059,0.1569),\ z_3^{(1)}=(0.4104,0.1872),$$

$$z_4^{(1)}=(0.3778,0.1808),\ z_5^{(1)}=(0.4371,0.2656),\ z_6^{(1)}=(0.3480,0.2000),$$

$$z_1^{(2)}=(0.3480,0.2042),\ z_2^{(2)}=(0.4059,0.1808),\ z_3^{(2)}=(0.3368,0.2386),$$

$$z_4^{(2)}=(0.5376,0.2656),\ z_5^{(2)}=(0.3778,0.2305),\ z_6^{(2)}=(0.3864,0.2158),$$

$$z_1^{(3)}=(0.4671,0.1772),\ z_2^{(3)}=(0.5071,0.1808),\ z_3^{(3)}=(0.3369,0.1772),$$

$$z_4^{(3)}=(0.4884,0.2071),\ z_5^{(3)}=(0.5675,0.2305),\ z_6^{(3)}=(0.4004,0.1275),$$

$$z_1^{(4)}=(0.4059,0.2259),\ z_2^{(4)}=(0.5428,0.1872),\ z_3^{(4)}=(0.4325,0.1275),$$

$$z_4^{(4)}=(0.3778,0.2305),\ z_5^{(4)}=(0.3097,0.1808),\ z_6^{(4)}=(0.4059,0.1772).$$

Step 2. Utilize the IFDOWA operator to derive the overall aggregated values $z_i(i=1,2,\cdots,6)$ of the alternatives $x_i(i=1,2,\cdots,6)$, where $\lambda=0.5$:

$$z_1=\text{IFDOWA}_{w^{(1)}}(z_1^{(1)},z_1^{(2)},z_1^{(3)},z_1^{(4)})=(0.4352,0.1859),$$

$$z_2=\text{IFDOWA}_{w^{(2)}}(z_2^{(1)},z_2^{(2)},z_2^{(3)},z_2^{(4)})=(0.4747,0.1767),$$

$$z_3=\text{IFDOWA}_{w^{(3)}}(z_3^{(1)},z_3^{(2)},z_3^{(3)},z_3^{(4)})=(0.3877,0.1722),$$

$$z_4=\text{IFDOWA}_{w^{(4)}}(z_4^{(1)},z_4^{(2)},z_4^{(3)},z_4^{(4)})=(0.4581,0.2202),$$

$$z_5=\text{IFDOWA}_{w^{(5)}}(z_5^{(1)},z_5^{(2)},z_5^{(3)},z_5^{(4)})=(0.4513,0.2273),$$

$$z_6=\text{IFDOWA}_{w^{(6)}}(z_6^{(1)},z_6^{(2)},z_6^{(3)},z_6^{(4)})=(0.3876,0.1737).$$

Step 3. Utilize the score function to calculate the scores $s(z_i)(i=1,2,\cdots,6)$ of overall aggregated values $z_i(i=1,2,\cdots,6)$ of the alternatives $x_i(i=1,2,\cdots,6)$:

$$s(z_1)=0.2493,\ s(z_2)=0.2980,\ s(z_3)=0.2155,$$

$$s(z_4)=0.2379,\ s(z_5)=0.2240,\ s(z_6)=0.2139.$$

Use the scores $s(z_i)(i=1,2,\cdots,6)$ to rank the alternatives $x_i(i=1,2,\cdots,6)$, we obtain

$$x_2\succ x_1\succ x_4\succ x_5\succ x_3\succ x_6.$$

5 Concluding

In this paper, we propose a new argument-dependent approach, based on entropy and similarity measure, to determine the OWA weights. we apply the IFDOWA operator to a multi-attribute group decision making problem and illustrate the effectiveness of the approach. It is worth noting that the results in this paper can be further extended to interval-valued intuitionistic fuzzy environment.

Acknowledgements. The work is supported by the National Natural Science Foundation of China (71171187, 71371107), the National Basic Research Program of China (2010CB731405), Ministry of Education Foundation of Humanities and Social Sciences (10YJC630269).

References

1. Ahn B.S.: Preference relation approach for obtain OWA operators weights. International Journal of Approximate Reasoning 47, 166-178 (2008)
2. Atanassov K.: Intuitionistic fuzzy sets. Fuzzy Sets and System 20(1), 87-96 (1986)
3. Atanassov K.: Intuitionistic fuzzy sets: Theory and Applications. Heidelberg: Physica-Verlag (1999)
4. Burillo P., Bustince H.: Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets. Fuzzy Sets and Systems 78, 305-316 (1996)
5. Chen S.M., Tan J.M.: Handling multi-criteria fuzzy decision making problems based on vague set theory. Fuzzy Sets and Systems 67(2), 163-172 (1994)
6. Emrouznejad A., Amin G.: Improving minimax disparity model to determine the OWA operator weights. Information Sciences 180, 1477-1485 (2010)
7. Filev D.P., Yager R.R.: On the issue of obtaining OWA operator weights. Fuzzy Sets and Systems 94, 157-169 (1998)
8. Hong D.H., Choi C.H.: Multicriteria fuzzy decision-making problems based on vague set theory. Fuzzy Sets and Systems 114, 103-113 (2000)
9. Li D.F., Cheng C.T.: New similarity measure of intuitionistic fuzzy sets and application to pattern recongnitions. Pattern Recognition Letters 23, 221-225 (2002)
10. Ngwenyama O., Bryson N.: Eliciting and mapping qualitative preferences to numeric rankings in group decision making. European Journal of Operational Research 116, 487-497 (1999)
11. O'Hagan M.: Aggregating template rule antecedents in real-time expert systems with fuzzy set. Proc. 22nd Ann. IEEE Asilomar Conf. on Signals, Systems and Computers Pacific Grove, pp. 681-689, CA(1988)
12. Szmidt E., Kacprzyk J.: Entropy for intuitionistic fuzzy sets, Fuzzy Sets and Systems 118(3), 467-477 (2001)
13. Wang Y., Lei Y.J.: A technique for constructing intuitionistic fuzzy entropy. Control and Decision 22(12), 1390-1394 (2007)
14. Wang Y.M., Parkan C.: A minimax disparity approach for obtain OWA operator weights. Information Sciences 175, 20-29 (2005)
15. Wei C.P.,Tang X.J.: An intuitionistic fuzzy group decision making approach based on entropy and similarity measures. International Journal of Information Technology and Decision Making 10(6), 1111-1130 (2011)
16. Wei C.P., Wang P.: Entropy, Similarity Measure of Interval-valued Intuitionistic Fuzzy Sets and Their Applications. Information Science 181(19), 4273-4286 (2011)

17. Xia M.M., Xu Z.S.: Some new similarity measures for intuitionistic fuzzy values and their application in group decision making. Journal of Systems Science and Systems Engineering 19(4), 430-452 (2011)
18. Xu Z.S.: Intuitionistic fuzzy aggregation operators. IEEE Transactions on Fuzzy Systems 15, 1179-1187 (2007)
19. Xu Z.S.: Dependent OWA operator. In: Torra, V., Narukawa, Y., Valls, A., DomingoFerrer, J.(eds.) MDAI 2006, LNCS (LNAI), 3885, pp. 172-178. Springer, Heidelberg (2006)
20. Xu Z.S.: An overview of methods for determining OWA weights. International Journal of Intelligent Systems 20(8), 843-865 (2005)
21. Xu Z.S., Da Q.L.: The uncertain OWA operator. International Journal of Intelligent Systems 17, 569-575 (2002)
22. Xu Z.S., Yager R.R.: Some geometric aggregation operators based on intuitionistic fuzzy sets. International Journal of General System 35(4), 417-433 (2006)
23. Yager R.R.: Quantifer guided aggregation using OWA operators. International Journal of Intelligent Systems 11(1), 49-73 (1996)
24. Yager R.R.: Families of OWA operators. Fuzzy Sets and Systems 59 125-148 (1993)
25. Yager R.R., Filev D.P.: Parameterized and-like and or-like OWA operators. International Journal of General Systems 22(3), 297-316 (1994)
26. Yager R.R.: On ordered weighted averaging aggregation operators in multicriteria decision making. IEEE Transactions on Systems, Man and Cybernetics 18, 183-190 (1988)
27. Yager R.R., Kacprczyk K.: The ordered weighted averaging operators: thery and applications. MA, USA: Kluwer Academic Publishers Norwell 275-294 (1997)
28. Zadeh A.: A computational approach to fuzzy quantifiers in natural languages. Computers and Mathematics with Applications 9, 149-184 (1983)
29. Zadeh A.: A computational theory of dispositions. International Journal of Intelligent Systems 2, 39-64 (1987)