Triadic and M-cycles Balance in Social Network

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Abstract: In this paper, we investigate the relation between triadic and m-cycles (*m*>3) motifs balance from theoretical implications, empirical statistics and numeric simulation. We suggest that m-cycles balance is consistent with triadic balance, the well-known structure balance theory proposed by Heider (1946). Finally, we use World War II hostile alliance relations to verify the real world situation.

I. Introduction

Social network theories and analytic methodologies are increasing attracting the attention of academic and industry researchers. Due to the essential function of social networks for analyzing the structure of whole social entities as well as a variety of theories explaining the patterns observed in these structures, in the past several decades, social network analysis has penetrated into multiple disciplines ranging from socio psychology to economics, management science, social physics, social computing and complex system theories. Networks are relational structures, and social networks represent structures of dyadic ties between social actors: Examples are friendship between individuals, alliances between firms, or trade between countries. The nature of networks leads to dependence between actors, and also to dependence between network ties.

With the booming of Web 2.0, online social networking sites produce unprecedented population together with real records of all their on-line behaviors, which leads to new exploitations and diverse modeling of human behaviors at Big Data era. For those increasing online complex networks, the emergence of global properties from local interactions is an interesting topic. Such large-scale networks are good examples of real world communities of interacting individuals in which local ties between individuals (like/dislike, friend/foe, support/against, cooperate/defect, trust/distrust etc.) give rise to a complex, multidimensional web of aggregated social behaviors. So far theoretical and empirical investigations are mostly at structural and topological level, however, the interactive relationships, or social ties [1] are often even more important than their topologies. Since interactive relationships mirror the complex social structure and collective behaviors, not just pure topology itself. From mathematical aspect, the social interaction relationships can be viewed as positive and negative social ties between individuals, and the social network can be described by signed graph.

As to signed relation, Heider (1946) first forwarded structure balance theory based on triad intentionally referring to the cognitive structures of an individual person [2]. Due to his seminal work, the notion of balance has been extensively studied by many mathematicians and psychologists [3]. Cartwright and Harary (1956) proposed that the definition of balance with formulation of signed graph may be used generally in describing configurations of many different networks [4], such as communication networks, power systems, sociometric structures, systems of orientations, or perhaps neural networks. Their cornerstone result states that a signed graph is balanced if and only if in each cycle (i.e. the closed paths beginning and ending on the same node) contains an even number of negative edges, otherwise is said to be negative. Based on this m-cycles balanced signed graph definition, Cartwright and Harary (1956) extended the original triads balance formulation (Heider, 1946). The definition of m-cycles balanced signed graph interpret that the potential source of tenses in the social network are number of negative edges for its each m-cycle.

From the standpoint of structuralism, one of the basic questions in modeling social networks is how the global properties of networks can be understood from local properties. The studies about transitivity in social networks propose that the local structure in social networks can be expressed by the triad census or triad count, the numbers of triads of any kinds. For (undirected) graphs, there are four triad types as shown in Fig.1.

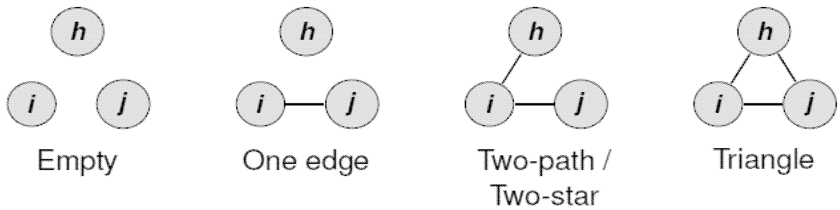

Fig.1 Four triad types on an undirected graph

The triad census brings information relevant to network, such as density, degree variance, and transitivity. For example, the well known small world property is derived from high local clustering coefficient and short average diameter [5]. Davis and Leinhardt (1972) proposed that social relations (social ties) could be tested on directed rather than undirected triad census, and forwarded 16 different types of directed triads (as shown in Fig.2) [6].

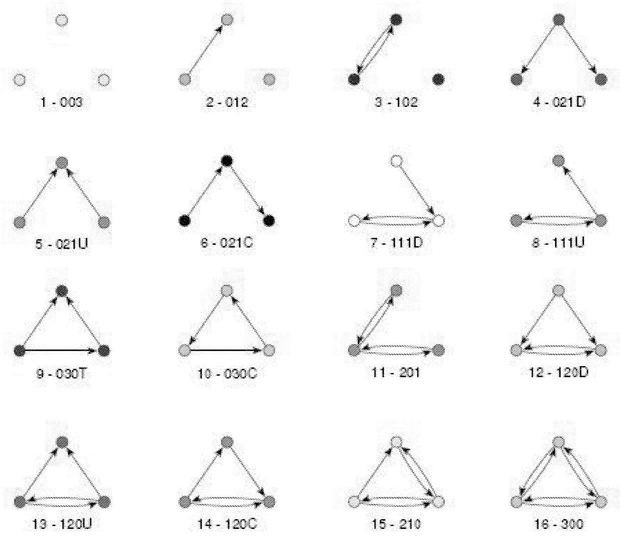

Fig.2 16 different types of directed triads [6].

The Davis and Leinhardt classification scheme describes each triad by a string of four elements: the number of mutual (complete) dyads within the triad; the number of asymmetric dyads within the triad; the number of null (empty) dyads within the triad; and a configuration code (U for up, D for down, C for cyclical and T for transitive) the triads which are not uniquely distinguished by the first three distinctions. For example, 120D refers the triad includes 1 mutual dyad, 2 asymmetric dyads, 0 null dyad and the down orientation; 021C refers the triad includes 2 asymmetric dyads, 1 null dyad and the cyclical orientation with no mutual dyad.

Triadic motifs become the most important building blocks for analyzing global social network properties. Based on triadic structure balance census, on-line signed social network global property has recently attracted some attention. Many works also addressed the inherent connection between network global emerging pattern and local triadic social structure balance by agent-based modeling [7-10].

While such triadic local properties are quite important, it is not clear if they capture all processes of substantive interest in social network, especially in signed network. Indeed, it has long been argued that local structure itself may be affected by large-scale network properties.

Comparing with the deeply studied triadic social structures (such properties include reciprocity, transitivity, clustering, status hierarchies and the degree distribution,) m-cycle problems are rarely addressed both theoretically and empirically. However, long-range cycles might be inarguably important for large-scale network properties. For instance, Willer (1999) [11] found that in negative exchange networks, long-range cycles can significantly affect the balance of even/odd paths between actors, and result in changing their payoffs, therefore change their incentives to trade. Some studies also found that long-range cycles play the role of maintaining reputation and trust transmission in social systems, serve as redundant conduits for the flow of goods and information [12, 13]. Individuals pass along reputation information about previous exchanges can successfully facilitate cooperation in repeated prisoner's dilemma games [14,15].

We argue that long range cycles might play complement role for our understanding the emerging global properties (or patterns) of network, which is ignored in most investigations.

With this motivation, in this study, we try to investigate the m-cycle balance in signed network from theoretical simulation and empirical study. The arrangement of the paper is as follows. Section 2 explains the 4-cycles conditional dependence social structure and it connection to social circuit model. In Section 3 we verify the connection and consistency between m-cycles balance and triadic balance by numeric simulation. In Section 4, we verify the consistency with World War II hostile alliance relations for testing. Section 5 gives our conclusions.

2. Social circuit model and m-cycles

Pattison and Robins (2002) suggested a family of partial conditional dependence models based on 3-paths, which include 4-cycles statistics in their full specification [16]. Snijders et al. (2006) [17] further proposed the social circuit assumption, formally as social circuit model. This model is based on the assumption that for four distinct nodes i, j, h, k social ties X_{ij} and X_{hk} are conditionally independent, given the rest of the graph. Actually, if $i = j$, it means h, k share the same neighbor (see Fig.3a), with high probability that h, k will build social tie. The transitive triads conditionally dependent (or Markov dependent) become the basic theoretical assumption which has been fully studied as above discussion. Triadic motif is also the basic building block for exponential random graph (Frank & Strauss, 1986)[18]. Here we consider the situation that if edges X_{ih}, X_{jk} exist as in Fig.3b, the possible tie between *i* and *j* implies that the four nodes i, j, h, k may be jointly included in a social setting, which then may affect the conditional probability of the tie between h and k. Thus the possible tie between *i* and *j* implies a 4-cycles in the graph under the social circuit assumption (see Fig.3b).

a. Markov model b. social circuit model

Fig. 3 Social circuit model and 4-cycles

The edgewise and dyad-wise shared partner statistics reflect tendencies toward transitive closure. The problem with the Markov specification is that the conditional log-odds of this edge increases linearly with the number of shared partners, which is a too strong dependence and leads to degeneracy of parameters estimation in exponential random graph model (ERGM), which may lead to convergence problems and a poor fit to empirical data [19] .

Based on social circuit theoretical assumption, with the concept of partial conditional independence of Pattison and Robins (2002) [16], the degeneracy problems are greatly improved. Social circuit dependence appears to reflect social processes underlying network formation better than simple Markov neighborhoods. Wide experiences have confirmed that these statistics can represent quite a large variety of observed social network.

Instead of just 4-cycles partial dependence, Butts (2006) [20] introduced a family of cycle statistics, which allow for the modeling of long range m-cycles (m>4) dependence or reciprocal path dependence extended from reciprocity dependence within exponential random graph framework. Based on four real world networks (Taro Exchange, Texas EMON, Friendship, 2000MIDs), the study suggests rather different underlying influences across the four networks.

However, what the relationship is it between triadic balance and m-cycle balance? This topic is rarely investigated till now. Next, we illustrate the connection between m-cycles balance and triads balance by numerical simulation based on global balance index (GBI).

3. The connection between m-cycles balance and triadic balance.

In this section, based on global balance index (GBI) [21], for m-cycles in a graph, by simulation we show that only if m-cycles is balanced, the corresponding decomposed local triangles are all balanced.

With the motivation of investigating the connection between m-cycles balance and triadic balance, for generic situation in signed network (the three types of social ties positive, negative and neural), here we investigate how influence signs "+" positive, "－" negative, and "0" neural change at the dyadic level affects the global (collective) balance state in the whole interpersonal network.

It is assumed that the interpersonal network tends toward higher balance (Heider, 1946), or to be evolved with the probabilities as shown in Eq.(1)

$$\begin{aligned} &\Pr[R_{ij}=0 \to R_{ij}=1 \bigcup R_{ij}=0 \to R_{ij}=-1] \approx 1 \\ &\Pr[R_{ij}=1 \to R_{ij}=0 \bigcup R_{ij}=-1 \to R_{ij}=0] \approx 0. \end{aligned} \quad (1)$$

Given a signed network, a fundamental problem is how to construct a dynamics of sign changes on the edges such that asymptotically the entire network is found on a perfectly balanced state. We use the GBI (Eq.(2)) to measure the network global balance level.

$$GBI = \sum_{i,j=1}^{n} (1 - s_i R_{ij} s_j)/2, \tag{2}$$

where the summation runs over all adjacent pairs of nodes, $s_i \in \{-1,+1\}, i = 1,...,n$ represents agent i's opinion, and R_{ij} represents the social tie signs which can be positive, negative and neutral, i.e. $R_{ij} \in \{+1,-1,0\}$.

Computing global balance means assigning "+1" or "-1" to each node in the network so as to minimize the GBI. GBI approximates to 0 means that interpersonal network reaches a global structure balance state. We randomly generate a 10×10 interpersonal network and 1×10 opinion vector. Fig.4. shows the dynamic signed influence network balance processes with using GBI as measure, we simulate 10 times and each time observe that the GBI reaches 0, i.e. the social network reaches a global network balance state within 1000 steps. Table 1 is the one of 10 times randomly generated initial signed social ties, Table 2 is the corresponding signed social ties after GBI reaches 0. Fig.5 is the graph corresponding to Table 2 adjacency signed matrix. We observe that for each m-cycles (3<=m<=10) in Fig.5 includes even number negative edges (green solid line). For example, the largest 10 cycles (i.e. 1-2-3-4-5-6-7-8-9-10-1) includes 6 negative edges and 4 positive edges. It is easily observed that any m-cycles (3<m<10) in Fig.5 are balanced, e.g. 1-2-3-4-5-6-7-8-9-1, 1-2-3-4-5-6-7-8-1, 1-3-5-6-7-9-1,8-10-2-5-6-8, 7-5-3-10-7. In addition, we find that all 3-cycles or triads are balanced, such as 1-3-4-1, 10-8-9-10, 3-7-9-3, 1-6-8-1, etc. Theoretically, it suggests that when the interpersonal network reaches a global structure balance state, based on global balance index (GBI), m-cycles balance (m>3) is consistent with triadic balance.

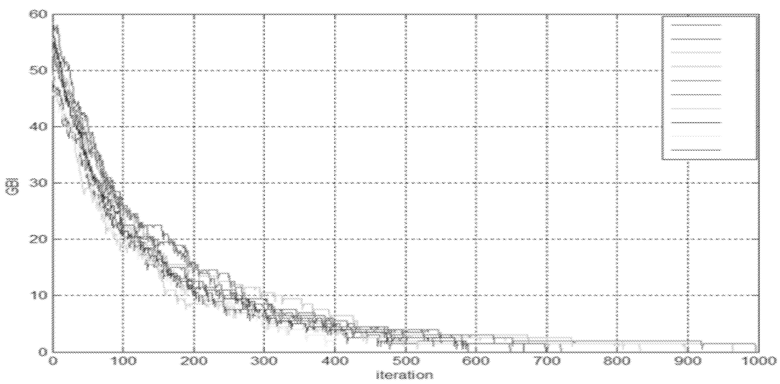

Fig. 4 Simulation results of GBI [22]

Table 1. Initial social ties matrix

0	-1	-1	0	-1	-1	0	1	-1	1
-1	0	0	1	-1	1	1	0	-1	0
1	0	0	-1	-1	1	-1	1	-1	1
1	0	0	0	1	0	1	1	-1	0
-1	1	0	-1	0	1	0	0	-1	1
-1	-1	-1	1	1	0	1	1	-1	-1
0	-1	0	1	0	-1	0	1	1	0
1	0	1	-1	1	0	1	0	0	-1
1	-1	1	-1	1	0	-1	0	0	-1
0	-1	-1	1	-1	1	1	1	-1	0

Table 2. The social ties matrix when GBI=0

1	1	1	1	1	-1	1	-1	1	-1
1	1	1	1	1	-1	1	-1	1	-1
1	1	1	1	1	-1	1	-1	1	-1
1	1	1	1	1	-1	1	-1	1	-1
1	1	1	1	1	-1	1	-1	1	-1
-1	-1	-1	-1	-1	1	-1	1	-1	1
1	1	1	1	1	-1	1	-1	1	-1
-1	-1	-1	-1	-1	1	-1	1	-1	1
1	1	1	1	1	-1	1	-1	1	-1
-1	-1	-1	-1	-1	1	-1	1	-1	1

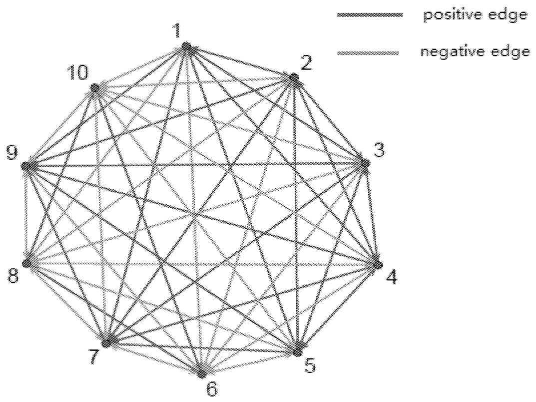

Fig.5 The interpersonal network when GBI=0

After the interpersonal network reaches the stable balance level (i.e. Table 2), we detect the local triadic interpersonal structure using R package sna [23]. Fig. 6 illustrates that code 300 (see Fig.2) is the only triadic structure remained, which according to [2,21] suggests that when GBI=0, local interpersonal relation attains structure stable state.

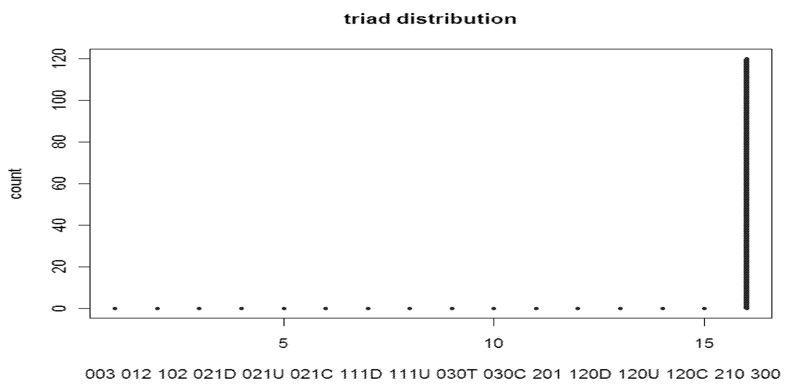

Fig. 6 16 types triadic distributions after GBI reach 0.

We also investigate the relation between the number of balanced m-cycles against the length of *m*-cycles, as illustrated in Fig.7. The figure shows that when *m*=5, the number of balanced *m*-cycles reaches maximum 252, the second is 210 when *m*=4 or 6.

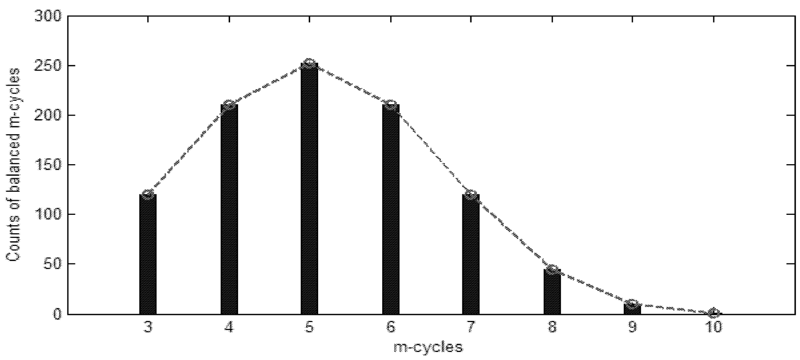

Fig.7 The number of balanced m-cycles vs. m-cycles length

As we simulate theoretically above, local triads balance is consistent with global m-cycles balance.

4. Empirical investigation

Next, we investigate the 3-cycles balance and m-cycles (m>4) balance distribution respect to major 15 states which were assembled in two opposing alliances and joined the World War I. The Allies (based on the Triple Entente of the United Kingdom, France and the Russian Empire) and the Central Powers of Germany and Austria-Hungary. Although Italy had also been a member of the Triple Alliance alongside Germany and Austria-Hungary, it did not join the Central Powers, as Austria-Hungary had taken the offensive against the terms of the alliance. These alliances were reorganized and expanded as more nations entered the war: Italy, Japan and the United States joined the Allies, and the Ottoman Empire and Bulgaria the Central Powers. A hostile alliance of adjacency matrix (where "+1" indicates alliance relation, "-1" denotes hostile relations) is shown in **Table 3**. (The state names are coded as digital numbers: Yugoslavia-1, Russia-2, *Bulgaria-3*, United Kingdom-4, Japan-5, Portugal-6, *Turkey-7*, Belgium-8, *Germany-9*, Greece-10, Romania-11, *Austria-Hungary-12*, France-13, Italy-14, and USA-15.)

Table 3. The adjacency matrix of hostile alliance in World War I

	1	**2**	***3***	**4**	**5**	**6**	***7***	**8**	***9***	**10**	**11**	***12***	**13**	**14**	**15**
1	1	1	−1	1	1	1	−1	1	−1	1	1	−1	1	1	1
2	1	1	−1	1	1	1	−1	1	−1	1	1	−1	1	1	1
3	−1	−1	1	−1	−1	−1	1	−1	1	−1	−1	1	−1	−1	−1
4	1	1	−1	1	1	1	−1	1	−1	1	1	−1	1	1	1
5	1	1	−1	1	1	1	−1	1	−1	1	1	−1	1	1	1
6	1	1	−1	1	1	1	−1	1	−1	1	1	−1	1	1	1
7	−1	−1	1	−1	−1	−1	1	−1	1	−1	−1	1	−1	−1	−1
8	1	1	−1	1	1	1	−1	1	−1	1	1	−1	1	1	1
9	−1	−1	1	−1	−1	−1	1	−1	1	−1	−1	1	−1	−1	−1
10	1	1	−1	1	1	1	−1	1	−1	1	1	−1	1	1	1
11	1	1	−1	1	1	1	−1	1	−1	1	1	−1	1	1	1
12	−1	−1	1	−1	−1	−1	1	−1	1	−1	−1	1	−1	−1	−1
13	1	1	−1	1	1	1	−1	1	−1	1	1	−1	1	1	1
14	1	1	−1	1	1	1	−1	1	−1	1	1	−1	1	1	1
15	1	1	−1	1	1	1	−1	1	−1	1	1	−1	1	1	1

We compute the counts of the *m*-cycle balanced motifs. The computation result is illustrated in **Fig.8**. With the simulation result in **Fig.7,** one interesting finding is that when *m*=5 or 6, the numbers of balanced *m*-cycles reaches maximum, when *m*=4, the network balanced counts reach the second maximum value. However, we still do not determine the reasons behind the same computing results. It is also obviously shown that the counts of balanced *m*-cycles decrease with the increase of *m*-cycles length. We also investigate the *m*-cycles balanced motifs distribution: 3-cycles or triadic motif balance is coherent with *m*-cycles motif balance. This result is same as illustrated in our numeric simulation except for the special case as *m*=7.

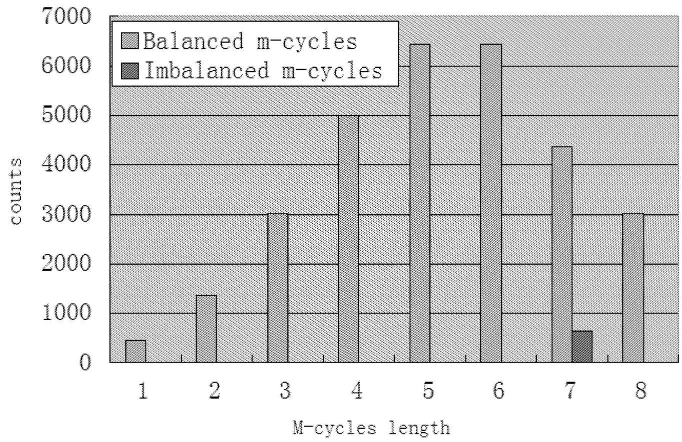

Fig 8. Balanced and imbalanced m-cycles distribution in the hostile alliance network of World War I

5. Conclusion

In this paper, we investigate the connection between triadic balance and *m*-cycles balance in a social network, from social structure aspects. We verify the connection and consistency between m-cycles balance and triadic balance by numeric simulation. This is not pure graph deduction or numerical simulation; we can find the specific theoretic ground, such as cooperation, indirect reciprocity, and social exchange. From social exchange point of view, the *m*-cycles has explicit Chain Generalized Exchange implication, e.g. the Kula Ring [24], one of the most cited examples of a generalized exchange process. The Kula means "the fundamental impulse to display, to share, to bestow and deep tendency to create social ties.

Our conclusion proves that long range exchange process chain balance is equal to local triadic balance, which is not contradicted to Heider's structure balance theory. This result shows that our social system might obey some basic mechanisms as the nature law, which takes the social relation as one medium to maintain normal operations (e.g. cooperation, economic exchange norm) of the social system. We argue that *m*-cycles balanced social structure (or social pattern) is such a mechanism preserved in the evolving processes of social interaction.

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