Hesitant Fuzzy Information Aggregation With A Prioritization Relationship Between Attributes

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Abstract—This paper proposes a method to determine weight vector of attributes with linear prioritization relationship and the assessment values are represented by hesitant fuzzy elements. The weight associated with an attribute depends upon the satisfaction of an alternative for the higher priority attribute. First, some hesitant fuzzy prioritized aggregation operators are defined and their desirable properties are discussed. These proposed operators can capture the prioritization phenomenon among the aggregated hesitant fuzzy elements. Then, we develop a multi-attribute decision-making (MADM) method based on the proposed operators in hesitant fuzzy environment. Finally, a practical example is provided to demonstrate the effectiveness of the developed method and make a comparative analysis on generalized hesitant fuzzy prioritized aggregation operators in decision-making.

Keywords—Multi-attribute decision-making, Hesitant fuzzy sets, Prioritized aggregation operators

I. INTRODUCTION

Torra and Narukawa [1] and Torra [2] defined the hesitant fuzzy set which allows the membership to have a set of possible values, some basic operations. Then, they studied its relationship with intuitionistic fuzzy set and fuzzy multisets. Afterwards, in order to aggregate the hesitant fuzzy information, Xia and Xu [3] proposed a series of aggregation operators under various situations and discussed the relationship among them. Then, they applied the developed aggregation operators to solve group decision-making problems with anonymity. Under the assumption that the values in all hesitant fuzzy elements are arranged in an increasing order and two hesitant fuzzy elements are of same length for comparison, Xu et al. [4], [5] and Chen et al. [6] defined a variety of distance measures, similarity measures and correlation measures, and then discussed their properties in detail. In addition, they proposed a number of hesitant ordered weighted distance measures and hesitant ordered weighted similarity measures. Xu et al. [7] developed some aggregation operators for hesitant fuzzy elements with the aid of quasi-arithmetic means, and gave two methods of determining the weight vectors with the aggregation operators based on the support degrees between the aggregated arguments and Choquet integral. Then, they proposed the corresponding decision-making method. Gu et al. [8] proposed the evaluation decision-making method for risk investment with hesitant fuzzy information based on the hesitant fuzzy weighted averaging (HFWA) operator, and demonstrated its effectiveness by an illustrative example. Xu and Xia [9] introduced the concepts of entropy and cross-entropy for hesitant fuzzy information, discussed their properties, and developed several measures formulas of entropy and cross-entropy. Then, they analyzed the relationship between them and similarity measure. Finally, they proposed two MADM methods based on the TOPSIS method.

From those results, we can know that hesitant fuzzy set is a very useful tool to deal with uncertainty and some MADM theories and methods have been developed under the hesitant fuzzy environment. However, above proposed MADM methods for HFES are under the assumption that the attributes are at the same priority level. They are characterized by the ability to trade off between attributes. For example, if $G_i$ and $G_k$ are two attributes with weight $\omega_i$ and $\omega_k$ respectively, in the decision-making method developed above, we can compensate for a decrease of $\Delta$ in satisfaction to attribute $G_i$ by gain $\frac{\Delta}{\omega_i}$ in satisfaction to attribute $G_k$. However, the attributes have different priority level in many real decision-making problems, so this kind of compensation between attributes is not feasible. A typical example is in the case of buying a car upon two attributes safety and cost. We give the assumption that attribute safety has a higher priority than attribute cost, it indicates that we are not willing to trade off satisfaction of attribute cost until perhaps we attain some level of satisfaction of attribute safety. Using the weighted aggregation operators to model the prioritized MADM is a effective method. Yager [10] showed that the prioritization of attributes can be modeled by using importance weights in which the weights associated with the lower priority attribute are related to the satisfaction of the higher priority attribute. To develop this concept, Yager [11] further proposed a prioritized averaging/scoring aggregation operator with a strict/weak priority order by means of the product t-norm, and the prioritized “and” operator and the prioritized “or” operator. Yager [12] proposed the prioritized OWA operator. Furthermore, taking DM’s requirements into account, Chen and Wang [13] and Wang and Chen [14] found the drawbacks of the method presented in Ref [10] by some numerical examples and suggested that the weights of the lower priority attribute depend on whether each alternative satisfies the requirements of all the higher priority attribute or not, proposed a generalized prioritized MADM method.
which overcome the drawbacks. Although previous researches have greatly developed the priority weighted MADM, there were still some limitations and drawbacks. Yan et al. [15] proposed aggregation operators to overcome the limitations of previous works, and showed the effectiveness and advantages of the proposed approach by comparative analysis with Ref [13], [14]. Wei and Tang [16] proposed generalized prioritized aggregation operators based on the WOWA operator. In intuitionistic fuzzy environment, Yu and Xu [17] proposed the intuitionistic fuzzy prioritized aggregation operator, gave a determining weighted method (IF-BUM) and developed the intuitionistic fuzzy prioritized OWA operator. Yu et al. [18] proposed the aggregation method for IVIFVs which has prioritization between attributes. Then, based on it and Ref [19], we define the hesitant fuzzy prioritized weighted aggregation operators in decision-making.

In this paper, we continue the research on the aggregation method for HFEs which has prioritized relationship between the aggregated arguments. The reminder of this paper is organized as follows. In section 2, we briefly review some basic knowledge. In section 3, we first propose a method for determining weight vector of the attributes for a linear order. The weight associated with an attribute depends upon the satisfaction of the higher priority attributes by modeling the prioritization between attributes. Then, based on it and Ref [19], we define the hesitant fuzzy prioritized weighted averaging (HFPWA) operator for hesitant fuzzy information, and developed corresponding approaches to solve the hesitant fuzzy MADM problems, in which the attributes are at different priority levels.

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Based on this weight-determined technics and Ref [19], we defined hesitant fuzzy prioritized weight averaging operator.

**Definition 3.1:** Let \( h_j(j = 1, 2, \ldots, n) \) be a collection of HFEs, and \( \omega = (\omega_1, \omega_2, \ldots, \omega_n) \) be the weight vector of \( h_j(j = 1, 2, \ldots, n) \), such that \( \omega_j = \frac{l_j}{\sum l_j}, T_1 = \{1\}, T_j = T_{j-1} \land h_j(j = 2, \ldots, n), l_1 = 1, l_j = \prod_{i=1}^{j-1} s(T_i)(j = 2, \ldots, n), \) and \( s(T_j) \) is the score value of \( T_j(j = 1, 2, \ldots, n) \). Then we define the hesitant fuzzy prioritized weighted average (HFPWA) operator as follows:

\[
HFPWA(h_1, h_2, \ldots, h_n) = \left( \frac{l_1}{\sum l_1} h_1 \oplus \frac{l_2}{\sum l_1} h_2 \oplus \cdots \oplus \frac{l_n}{\sum l_1} h_n \right)^\frac{1}{h}
\]

(3.5)

Analogous to Ref [19], it can be easily proved that HFPWA operator has the Idempotency, Monotonicity and Boundedness.

**Theorem 3.1:** Let \( h_j(j = 1, 2, \ldots, n) \) be a collection of HFEs, and \( \omega = (\omega_1, \omega_2, \ldots, \omega_n) \) be the weight vector of \( h_j(j = 1, 2, \ldots, n) \), such that \( \omega_j = \frac{l_j}{\sum l_j} \), \( T_1 = \{1\}, T_j = T_{j-1} \land h_j(j = 2, \ldots, n), l_1 = 1, l_j = \prod_{i=1}^{j-1} s(T_i)(j = 2, \ldots, n) \), and \( s(T_j) \) is the score value of \( T_j(j = 1, 2, \ldots, n) \). Then their aggregated value by using the HFPWA operator is also a HFE, and

\[
GHFPWA_\lambda(h_1, h_2, \ldots, h_n) = \left( \frac{l_1}{\sum l_1} h_1 \lambda \oplus \frac{l_2}{\sum l_1} h_2 \lambda \oplus \cdots \oplus \frac{l_n}{\sum l_1} h_n \lambda \right)^\frac{1}{\lambda}
\]

(3.7)

then GHFPWA is called a generalized hesitant fuzzy prioritized weighted average operator.

If \( \lambda = 1 \), then the GHFPWA operator becomes the HFPWA operator.

According to Ref [19], It can be proved that the GHFPWA operator has followed the properties:

1. **Idempotency:** If all HFE \( h_j(j = 1, 2, \ldots, n) \) are equal, i.e. \( h_j = h \), for all \( j \), then \( GHFPWA_\lambda(h_1, h_2, \ldots, h_n) = h \).

2. **Monotonicity:** Let \( h_j(j = 1, 2, \ldots, n) \) be a collection of HFEs, \( h_j'(j = 1, 2, \ldots, n) \) is also a collection of HFEs, and \( h_j \leq h_j' \), then \( GHFPWA_\lambda(h_1, h_2, \ldots, h_n) \leq GHFPWA_\lambda(h_1', h_2', \ldots, h_n') \).

3. **Boundedness:** Let \( h_j(j = 1, 2, \ldots, n) \) be a collection of HFEs, \( h_- = \min_j h_j \), \( h_+ = \max_j h_j \), then \( h_- \leq GHFPWA_\lambda(h_1, h_2, \ldots, h_n) \leq h_+ \).

**Lemma 3.1:** [20][21] Let \( x_j > 0, \lambda_j, j = 1, 2, \ldots, n \), and \( \lambda_j, j = 1, 2, \ldots, n \), then \( \prod_{j=1}^{n} x_j^{\lambda_j} \leq \sum_{j=1}^{d} \lambda_j x_j \) with equality if only if \( x_1 = x_2 = \cdots = x_n \).

**Theorem 3.2:** Let \( h_j(j = 1, 2, \ldots, n) \) be a collection of HFEs, and \( \lambda > 0, \omega = (\omega_1, \omega_2, \ldots, \omega_n) \) the weight vector of \( h_j(j = 1, 2, \ldots, n) \), such that \( \omega_j = \frac{l_j}{\sum l_j}, T_1 = 1, T_j = T_{j-1} \land h_j(j = 2, \ldots, n), l_1 = 1, l_j = \prod_{i=1}^{j-1} s(T_i)(j = 2, \ldots, n), \) and \( s(T_j) \) is the score value of \( T_j(j = 1, 2, \ldots, n) \). If

**B. Generalized hesitant fuzzy prioritized aggregation operators**

Based on the definitions and the properties of the HFPWA operator in subsection A, we propose some generalised hesitant fuzzy prioritized aggregation operators, and give some properties.

**Definition 3.2:** Let \( h_j(j = 1, 2, \ldots, n) \) be a collection of HFEs, \( \lambda > 0, \omega = (\omega_1, \omega_2, \ldots, \omega_n) \) the weight vector of \( h_j(j = 1, 2, \ldots, n) \), such that \( \omega_j = \frac{l_j}{\sum l_j}, T_1 = 1, T_j = T_{j-1} \land h_j(j = 2, \ldots, n), l_1 = 1, l_j = \prod_{i=1}^{j-1} s(T_i)(j = 2, \ldots, n), \) and \( s(T_j) \) is the score value of \( T_j(j = 1, 2, \ldots, n) \). If
If
\[ QHFPA(h_1, h_2, \ldots, h_n) = f^{-1}\left( \sum_{i=1}^{l_1} \frac{l_i}{\sum_{i=1}^{n}} f(h_i) \oplus \left( 1 - \prod_{j=1}^{n} (1 - f(\gamma_j)) \right) \sum_{i=1}^{l_2} \frac{l_i}{\sum_{i=1}^{n}} f(h_i) \oplus \cdots \oplus \left( 1 - \prod_{j=1}^{n} (1 - f(\gamma_j)) \right) \sum_{i=1}^{l_n} \frac{l_i}{\sum_{i=1}^{n}} f(h_i) \right) \]
then \(QHFPA\) is called a quasi hesitant fuzzy prioritized weighted average operator, where \(f : [0, 1] \rightarrow [0, 1]\) is a strictly continuous monotonic function.

If \(f(x) = x\), then the \(QHFPA\) operator becomes the \(HFPWA\) operator.

If \(f(x) = x^\lambda, \lambda > 0\), then the \(QHFPA\) operator becomes the \(GHFPWA\) operator.

In fact, based on the ordered modular averages (OMAs) [22], we can further generalize the hesitant fuzzy prioritized operator as follows:

**Definition 3.4:** Let \(h_j(j = 1, 2, \ldots, n)\) be a collection of HFEs, and \(\omega = (\omega_1, \omega_2, \ldots, \omega_n)\) the weight vector of \(h_j(j = 1, 2, \ldots, n)\), such that \(\omega_j = \frac{l_j}{\sum_{i=1}^{n}}\), \(T_1 = 1, T_j = T_{j-1} \land h_j(j = 2, \ldots, n), l_1 = 1, l_j = \prod_{i=1}^{j} s(T_i)(j = 2, \ldots, n)\), and \(s(T_j)\) is the score value of \(T_j(j = 1, 2, \ldots, n)\). If
\[ HFPMWA(h_1, h_2, \ldots, h_n) = \sum_{i=1}^{l_1} \frac{l_i}{\sum_{i=1}^{n}} f_1(h_i) \oplus \sum_{i=1}^{l_2} \frac{l_i}{\sum_{i=1}^{n}} f_2(h_i) \oplus \cdots \oplus \sum_{i=1}^{l_n} \frac{l_i}{\sum_{i=1}^{n}} f_n(h_i) \]
then \(HFPMWA\) is called a hesitant fuzzy prioritized modular weighted average operator, where \(f_i : [0, 1] \rightarrow [0, 1](i = 1, 2, \ldots, n)\) is a strictly continuous monotonic function.

If \(f_i(x) = x(i = 1, 2, \ldots, n)\), then the \(HFPMWA\) operator becomes the \(HFPWA\) operator.

**IV. AN APPROACH TO MULTI-ATTRIBUTE DECISION-MAKING UNDER HESITANT FUZZY ENVIRONMENT**

In this section, we utilize the proposed hesitant fuzzy prioritized aggregation operators to solve group decision-making problems under hesitant fuzzy environment. In a group decision-making problem, suppose \(X = \{x_1, x_2, \ldots, x_m\}\) is the set of alternatives, let \(G = (G_1, G_2, \ldots, G_n)\) be a collection of attributes and there is a prioritized relation between these attributes expressed by the linear ordering \(G_1 \succ G_2 \succ \cdots \succ G_n\), indicate attribute \(G_j\) has a higher priority \(G_k\), if \(j < k\). If decision makers provide all the possible evaluated values under the attribute \(G_j\) for the alternative \(x_i\) with anonymity, these values can be considered as a hesitant fuzzy element \(h_{ij}\). In the case where two decision makers provide the same value, then the value emerges only once in \(h_{ij}\). Suppose that the decision matrix \(H = (h_{ij})_{m \times n}\) is the hesitant fuzzy decision matrix, where \(h_{ij}(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)\) is in the form of HFEs. Then, based on the generalized hesitant fuzzy prioritized aggregation (GHFPWA and QHFPWA) operators, we give a method for group decision-making with hesitant fuzzy information, which involves the following steps:

**Step 1.** Calculate the weights \(\omega_{ij}\) of \(h_{ij}(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)\), as follows:
\[ T_{i1} = \{1\}, i = 1, 2, \ldots, m, \]
\[ T_{ij} = T_{i,j-1} \land h_{ij}(i = 1, 2, \ldots, m, j = 2, \ldots, n) \]
\[ l_{i1} = 1, i = 1, 2, \ldots, m, \]
\[ l_{ij} = \prod_{k=1}^{j} s(T_{ik})(i = 1, 2, \ldots, m, j = 2, \ldots, n), \]
\[ \omega_{ij} = \frac{l_{ij}}{\sum_{j=1}^{m}} (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n). \]

**Step 2.** Aggregate the hesitant fuzzy values \(h_{ij}(j = 1, 2, \ldots, n)\) by using the hesitant fuzzy prioritized extension, denoted by \(\Theta\), then
\[ h_i = \Theta(h_{i1}, h_{i2}, \ldots, h_{in}), i = 1, 2, \ldots, m. \]
\(\Theta\) can be someone of the GHFPWA operator and the QHFPWA operator.

**Step 3.** Calculate the scores \(s(h_i)(i = 1, 2, \ldots, m)\) of the overall hesitant fuzzy preference values \(h_i(i = 1, 2, \ldots, m)\) and rank them.

**Step 4.** Rank all the alternatives \(A_i(i = 1, 2, \ldots, m)\) and select the best one(s) accordingly with \(s(h_i)(i = 1, 2, \ldots, m)\).

**Step 5.** End.

**V. PRACTICAL EXAMPLE**

Working to strengthen academic education and promoting the building of teaching body, the school of management in a Chinese university wants to recruit overseas outstanding faculties. This program has been raised great attention. University president \(e_1\), dean of management school \(e_2\), and human resource officer \(e_3\) sets up the panel of recruitment to take the whole responsibility for this program. They have made strict evaluation for 5 candidates \(x_i(i = 1, 2, 3, 4, 5)\) from four aspects, namely morality \(G_1\), research capability \(G_2\), teaching skill \(G_3\), education background \(G_4\). In addition, this program is in strict accordance with the principle of combine ability with political integrity. The prioritization relationship for attributes is shown as: \(G_1 \succ G_2 \succ G_3 \succ G_4\). The five candidates \(x_i(i = 1, 2, 3, 4, 5)\) are to be evaluated by the three decision makers under the above four attributes with anonymity, and construct the hesitant decision matrix \(H = (h_{ij})_{5 \times 4}\), which is shown in Table 1.
Table 1: Hesitant fuzzy decision matrix H

<table>
<thead>
<tr>
<th></th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$G_3$</th>
<th>$G_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>${0.4, 0.5, 0.7}$</td>
<td>${0.5, 0.8}$</td>
<td>${0.6, 0.7, 0.9}$</td>
<td>${0.5, 0.6}$</td>
</tr>
<tr>
<td>$x_2$</td>
<td>${0.6, 0.7, 0.8}$</td>
<td>${0.5, 0.6}$</td>
<td>${0.4, 0.6, 0.7}$</td>
<td>${0.4, 0.5}$</td>
</tr>
<tr>
<td>$x_3$</td>
<td>${0.6, 0.8}$</td>
<td>${0.2, 0.3, 0.5}$</td>
<td>${0.4, 0.6}$</td>
<td>${0.5, 0.7}$</td>
</tr>
<tr>
<td>$x_4$</td>
<td>${0.5, 0.6, 0.7}$</td>
<td>${0.4, 0.5}$</td>
<td>${0.8, 0.9}$</td>
<td>${0.3, 0.4, 0.5}$</td>
</tr>
<tr>
<td>$x_5$</td>
<td>${0.6, 0.7}$</td>
<td>${0.5, 0.7}$</td>
<td>${0.7, 0.8}$</td>
<td>${0.2, 0.3, 0.4}$</td>
</tr>
</tbody>
</table>

Then, in order to get the optimal result, we make an example of the GHFPWA operator ($\lambda = 1$) to develop an approach to MADM problem under hesitant fuzzy information, the main step is described as following:

**Step 1.** Utilize (4.10) and (4.11) to calculate the values of $\omega_{ij}(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ as follow:

$$
\omega_{ij} = \begin{bmatrix}
0.4339 & 0.2820 & 0.1833 & 0.1008 \\
0.5132 & 0.2822 & 0.1411 & 0.0635 \\
0.6734 & 0.2245 & 0.0783 & 0.0274 \\
0.5769 & 0.2596 & 0.1168 & 0.0467 \\
0.4836 & 0.2901 & 0.1741 & 0.0522
\end{bmatrix}
$$

**Step 2.** Aggregate all hesitant fuzzy values $h_{ij}(j = 1, 2, \cdots, n)$ by using the (GHFPWA) operator to derive the overall hesitant fuzzy values $h_i(i = 1, 2, 3, 4, 5)$ of the candidates $x_i$. If $\lambda = 1$, take alternative $x_1$ for an example, we have

$$
h_1 = GHFPWA_1(h_{11}, h_{12}, h_{13}, h_{14}) = GHFPWA_1(\{0.4, 0.5, 0.7\}, \{0.5, 0.8\}, \{0.6, 0.7, 0.9\}, \{0.5, 0.6\}) = \bigcup_{\gamma_{11} \in h_{11}, \gamma_{12} \in h_{12}, \gamma_{13} \in h_{13}, \gamma_{14} \in h_{14}} \left\{1 - \prod_{j=1}^{4} (1 - \gamma_{1j})^{\omega_{ij}}\right\}
$$

In Table 2, we can see that the sort results of $x_1, x_2, x_3, x_4, x_5$ will change with the increase of $\lambda$.

![Table 2: the sequence of alternatives]

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$s(x_1)$</th>
<th>$s(x_2)$</th>
<th>$s(x_3)$</th>
<th>$s(x_4)$</th>
<th>$s(x_5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GHFPWA_1</td>
<td>0.6286</td>
<td>0.6535</td>
<td>0.6248</td>
<td>0.6501</td>
<td>0.6401</td>
</tr>
<tr>
<td>GHFPWA_2</td>
<td>0.6329</td>
<td>0.6736</td>
<td>0.6306</td>
<td>0.6100</td>
<td>0.6486</td>
</tr>
<tr>
<td>GHFPWA_3</td>
<td>0.6420</td>
<td>0.6425</td>
<td>0.6411</td>
<td>0.6208</td>
<td>0.6536</td>
</tr>
<tr>
<td>GHFPWA_5</td>
<td>0.6514</td>
<td>0.6460</td>
<td>0.6497</td>
<td>0.6328</td>
<td>0.6583</td>
</tr>
<tr>
<td>GHFPWA_7</td>
<td>0.6605</td>
<td>0.6534</td>
<td>0.6621</td>
<td>0.6378</td>
<td>0.6669</td>
</tr>
<tr>
<td>$x_1 &gt; x_3 &gt; x_2 &gt; x_4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_2 &gt; x_1 &gt; x_3 &gt; x_4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_3 &gt; x_2 &gt; x_1 &gt; x_4$</td>
<td></td>
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</tbody>
</table>

Analogously, we can perform the same analysis on the QHFPWA operator, thereby, according to the different aggregated requirements under hesitant fuzzy environment, we can choose the appropriate generalized prioritized aggregation operator in solving MADM problem, which takes into account prioritization among attributes.

**VI. Conclusion**

In this paper, we investigate the hesitant fuzzy MADM problems in which the attributes are in different priority level. Then, based on the idea of prioritized aggregation operator [2], [5], [22] and the hesitant fuzzy prioritized aggregation operator proposed by Ref [19], we propose a different method to determine the weight vectors associated with the prioritized relationship of the aggregated arguments. In addition, we define the HFPWA operator based on the proposed method. Moreover, we develop some generalized hesitant fuzzy prioritized aggregation operator (GHFPWA,QHFPWA,HFPMAW) and investigate some of their desirable properties in detail. To reflect the priority level of the aggregated arguments, we apply these proposed generalized prioritized aggregation operators to develop a MADM method that take into account prioritization among attributes. Finally, an example is given to illustrate the effectiveness of decision-making methods, and we make further analysis on the sequence of alternative by the different generalized hesitant fuzzy prioritized aggregation operators. It is worth noting that the results of this paper can be extended to the interval hesitant fuzzy environment.

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**References**


