Polarization and Non-Positive Social Influence: A Hopfield Model of Emergent Structure

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ABSTRACT

The authors study patterns about group opinions in a group-based society by considering social influence. They classify three types of social influence: positive, neutral, and negative from the perspective of social identity, and investigate to what extent the non-positive social influence leads to group opinion polarization based on the Hopfield network model. Numerical simulations show that opinion in a group-based society would self-organize into bi-polarization pattern under the condition of no imposing external intervention, which is entirely different from the result of drift to an extreme polarization dominant state with single homogenous influence. These results are explained in the study and the authors show that opinions polarization in a group is coexisted with local structure balance.

Keywords: *Bi-Polarization, Group Opinions, Group-Based Societies, Hopfield Network Model, Local Structure Structure Balance, Opinion Dynamics, Social Influence*

INTRODUCTION

Almost all social interactions are, at least in part, shaped by opinions. Some opinions are ultimately evolved into acceptable behaviors such as what we deem as social norms. They not only affect our daily lives but also shape our political participation and attitudes. Towards a specific object or event, human collective opinions display different patterns, e.g. consensus, polarization and diversity with a variety of factors, such as mutual influence, the external intervention etc, which had been long studied by sociologists and become an important research field among multiple disciplines. As a matter of fact, opinion dynamics has been widely studied in management science (Simon, 1954), social psychology (French, 1956; Latane, 1981; Friedkin, 1998), economics (Blume & Durlauf, 2004), sociophysics (Stauffer, 2005; Galam, 2007), systems science (Hummel, 1996), statistic physics (Sznajd-Weron & Sznajd, 2000), etc. Recently, with the booming of Internet and advances in information technologies, the topic is maturing into the spotlight in computer science (Leskovec, Huttenlocher, & Kleinberg, 2010; Conover, Ratkiewicz et al., 2010), etc.

In this paper, we focus on the impacts of social influence toward group opinion, investigate to what extend that social influence can

DOI: 10.4018/jkss.2012070102

affect group opinion formation. We make a clear classification about three types of social influence as positive, neutral and negative based on social identity theory. Furthermore, we study the relation of polarization and non-positive influence through Hopfield network model simulation.

The rest of the paper is organized as follows: according to social identity, we discuss three kinds of social influence implications and make a clear classification. Structure balance theory and 16 types of triadic structure are referred. First, we extend Hopfield model implication and assign three types of discrete social influence weights into the model. By adding three types of social influence into Hopfield model, we study the relationship between opinion bi-polarization and non-positive social influence. Moreover, we analyze the local triad distribution before and after group opinions polarization, and discuss some interesting social-psychological implications about local level signed social structure balance and global pattern emergence. Finally we discuss our concluding remarks.

SOCIAL IDENTITY, SOCIAL INFLUENCE, SIGNED LINK WITH LOCAL COMMUNITY

Classification of Social Influence Based on Social Identity

Social influence occurs when an individual's thoughts, feelings or actions are affected by other people. One of the important principles underlying group opinion polarization is social influence toward individuals' behavior. Extensive research shows that social influence may trigger individuals to revise their estimates (Mason et al., 2007) and change their attitudes. When individuals observe attitudes or opinion of others, they follow the wisdom of crowds (Mannes, 2009), and herding effects may have pressure on their opinions formation. The effects are widely studied in many domains ranging from cognitive neuroscience to economics (Raafat et al., 2009; Burke et al., 2010).

Besides homogeneous attraction, both heterogeneous repulsive and neutral attitudes effects among agents are also important toward individuals' behavior in social systems. Herding effects only partially explain one aspect of collective behaviors, individuals in a group act together without planned direction, based on indefinite individuals' social identity.

In this paper we concern about what the opinion dynamics pattern will be if consider the positive attractive homogeneous, non-positive repulsive and neutral impact. At first we distinguish three different kinds of social influence from social identity perspective.

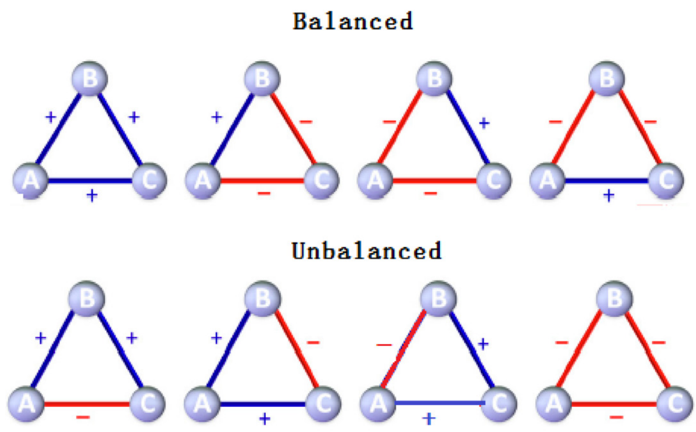
Social identity theory suggests that individuals have the self-concept identity derived from the perceived membership of social groups. Individuals are likely to display favoritism among ingroup and disapproval among outgroup. In other words, individuals often display positive attitude toward ingroup members, while negative toward outgroup ones. For example in the case of voting and debating about distribution of national income, different classes may have different interests and political tendencies, individuals will favor ingroup and against outgroup opinions or stands.

In many real situations, negative repulsive impact among social groups is an important ingredient while it has been barely focused together with positive attractive influence behavior in studies.

Generally to say, from the social influence point of view, three types of impact run through the whole processes of group decision-making especially in voting. One is positive influence among ingroup members; this kind of social force accelerates ingroup opinion convergence.

The second one is negative social impact which may block the formation of consensus among different outgroups. Individuals within different groups find it difficult to gain the agreement during group decision making even under the pre-condition that they share the same initial opinions. Since different groups have different social group unified interests, emotions, behaviors and value orientations, then they act differently. Such kind of impact for

Figure 1. Balanced and unbalanced triadic relationship ("+" denotes friendship, " – " denotes enemy relation (Facchetti, Iacono, & Altafini, 2011)



individuals' opinions selection is regarded as heterogeneous repulsion. The use of both positive and negative interactions in social systems has been previously introduced to coalitions study among several countries (Galam, 1996).

Except positive and negative influence, we also observe individuals who seldom care others and share no commons. As a type of special individuals' attitudes, in which the individuals might not belong to any labeled subgroup, members in the group have no common social identity, no firm stand about some opinions and are in a state of neither fish nor fowl.

Then in this paper we introduce the third one, unsocial phenomena as a type of special individuals' attitude, in which the individuals do not belong to any labeled subgroup. Members with such kind of attitude have no common social identity, no firm position about some social opinions and in a state of neither fish nor fowl.

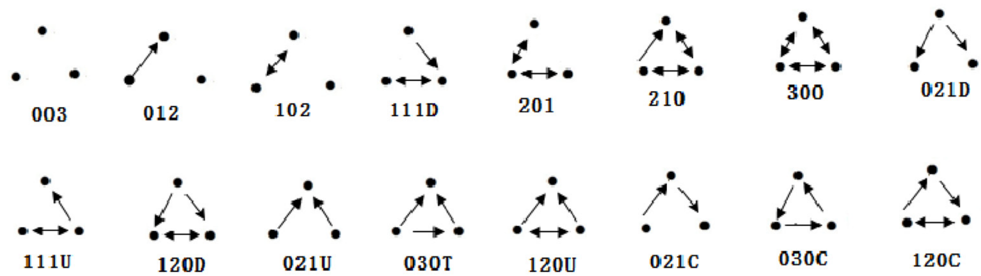
Structure Balance

As one of the important classic social theories, structure balance suggests that some social relationships are more stable than others (Cartwright & Harary, 1956; Davis & Leinhardt, 1972). It focuses on triadic relationships such as friendship and antagonism. In graphs, we use signed edges represent friendship/hostile relationship among individuals. Structure balance theory affirms that signed social networks tend to be organized so as to avoid tense or nervous situations. That follows the common principles that "the friend of my friend is my friend," "the enemy of my friend is my enemy," "the friend of my enemy is my enemy." Figure 1 illustrates the balanced and unbalanced triadic relationship.

Holland and Leinhardt (1970) addressed that classic balance theory offers a set of simple local rules for relational change and classified local triadic motifs into 16 types, according to mutual reciprocity, asymmetry relation, non-relationship between pairs, where Code 300 triad relation corresponding to structure balance under the condition of the triad product signs satisfies "+", which illustrated in Figure 2.

The booming of online social networking enable large-scale individuals interacting possible, and those ties between users (friend/enemy, trust/distrust, like/hate etc.) give rise to a complex multiplex web of aggregated social behavior. Some related experimental researches show that the global levels of balance of very large online social networks are indeed extremely balanced (Leskovec, Huttenlocher, & Kleinberg, 2010; Szell, Lambiotte, & Thurner, 2010; Facchetti, Iacono, & Altafini, 2011).

Figure 2. 16 types of triad distributions in classic structure balance theory (Holland & Leinhardt, 1970)



In this paper, we focus on the intrinsic relation between group polarization and social influence. Then we regard the interpersonal relation as shown in Figure 2 as social influence. Next, we discuss group polarization processes based on Hopfield network model.

HOPFIELD NETWORK MODEL

Macy et al. (2003) presented a Hopfield model to describe group polarization problems, with considering individual decision making dimensions, social ties signs, strengths and culture dissemination theories, etc. The mechanism of their modeling is as follows,

$$P_{is} = \frac{\sum_{j=1}^{N} I_{ij} s_j}{N}, j \neq i, \qquad (1)$$

for each individual i, the cumulative social pressure for her/him to choose s_i is denoted as Equ. (1), where $s_i = \pm 1$ represent binary voting opinions, N is group size, I_{ij} is the social influence that individual $j (j \neq i)$ imposed to i, matrix I is named social influence matrix.

Comparing with Macy et al. (2003), in this study, with the motivation of investigating the relationship between non-positive social influence and group opinions polarization, instead the assigning of continuous values between -1 and +1 to I_{ij}, we assign three discrete values -1, +1, 0 to I_{ij} to indicate the three types of social influence. Individuals (agents) are influenced by others and also influence others, as conditioned by the valence of the social influence tie I_{ij}, where $I_{ij} \in \{+,-,0\}$ listed and explained as follows:

1) "+" denotes the positive homogeneous social influence,
2) "$-$" stands for xenophobia, antagonistic, negative social influence,
3) "0" represents unsocial attitudes influence.

Furthermore, if we consider the external intervention, i.e., the influence for individuals opinion comes from other out-group impact), then replace Equ.(1) with Equ. (2) and obtain the logistic form, which is named cumulative social pressure.

$$\tau_{is} = \frac{v_s}{1 + e^{-KP_{is}}} + (1 - v_s) X_{i.} \qquad (2)$$

where v_s is used to trade off the internal and external group influence for individual i opinion, K is the size of opinions dimension. Given a randomly selected threshold π, if $\tau_{is} \geq \pi$, individual i chooses +1 (support), else chooses -1 (oppose). Equ. (3) describes the update of influence processes of individual j to i ($j \neq i$).

$$I_{ij}(t+1) = I_{ij}(t)(1-\lambda) + \frac{\lambda}{K}\sum_{k=1}^{K} s_{jk}(t)s_{ik}(t), j \neq i, \quad (3)$$

where t is the time step, λ is an adjustable parameter between 0 and 1.

SIMULATION

The algorithm of implementing Hopfield network social influence processes are listed as below:

Step 1: Let $t = 0$, given v_s, λ.

Initialize each voter k-th dimension opinion $s_{ik}(0) = \pm 1$, $k = 1,...,K; i = 1,...,N$, Randomly generate each pair of voters social influence I_{ij}.

Step 2: $t = t + 1$, compute social pressure P_{is} by Equ.(1), cumulative social pressure τ_{is} by Equ. (2) for each agent i, and randomly generate π

if $\tau_{is} > \pi$

$s_{ik}(t) = 1$

else

$s_{ik}(t) = -1$

Step 3: Compute social influence update by Equ.(3), for a given small positive real number ε,

if $\max_{i \neq j} | I_{ij}(t) - I_{ij}(t-1) | < \varepsilon$, stop

else go to Step 2.

SIMULATION RESULT ANALYSIS

In this section, we first explain the results of our simulation implemented by MATLAB, then we use R package for triad distribution analysis.

Bi-Polarization and Non-Positive Social Influence

We take the test by setting $N = 100$, $K = 5$, $\varepsilon = 0.01$ and $\lambda = 0.5$. We run 100 times for average. Figure 3(a) shows the group initial random opinions states when each agent i face K-dimension decision making (before self-organizing polarization). Figure 3(b) illustrates the group bi-polarization state under the condition of no imposing external influence ($v_s = 1$) and with three types of influence. We can observe that two patterns appear after group polarization, i.e., one pattern is (+1, +1, -1, -1, -1), i.e., (white, white, black, black, black) (marked by V_1), the other is (-1, -1, +1, +1, +1), i.e., (black, black, white, white, white) (represented by V_2). The ratio of the 2-pattern size is approximate to 1:1.

The relationship between exogenous intervention parameter v_s to group polarization is as shown in Figure 4. We can see that when $v_s = 1$ (no external intervention to the group interaction processes), the ratio of $V_1 / (V_1 + V_2)$ is approximate to 0.5. However, the fifty to fifty well matched equilibrium will be destroyed with a little cut off v_s. In other words, external intervention will lead to majority pattern appeared. In particular, when $v_s = 0.5$, group opinion is evenly affected by external and internal factors, we observe the group consensus appears, i.e., $V_1 / (V_1 + V_2)$ is approximate to 1, the pattern V_2 nearly disappears.

It is clearly suggested that, under the condition of imposing external intervention, the group reaches majority or consensus pattern. With no exogenous impact, the group evolves into bi-polarization state in the end.

Figure 3. Group opinion before and after polarization under the condition of imposing three types of social influence. (We generate $N \times K$ matrix which N denotes group size, K denotes the numbers of options, N=100, K=5)

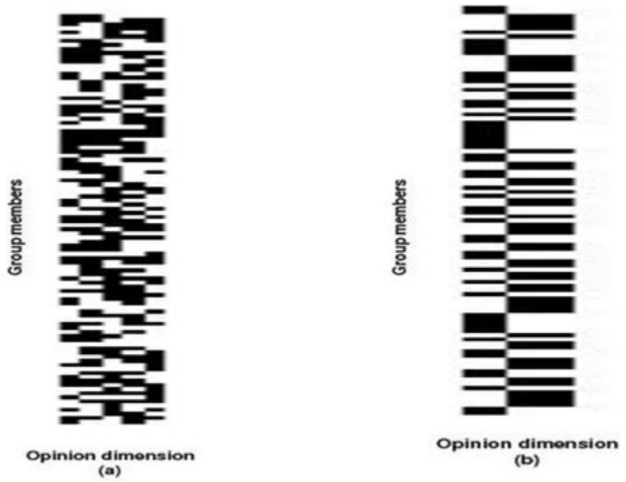


Figure 4. Exogenous intervention impact on group polarization (In Equ. (2) we adjust v_s from 1 to 0.5 i.e., increasing the out-group intervention from 0 to 0.5, through this way we can observe the exogenous effect on group opinions polarization)

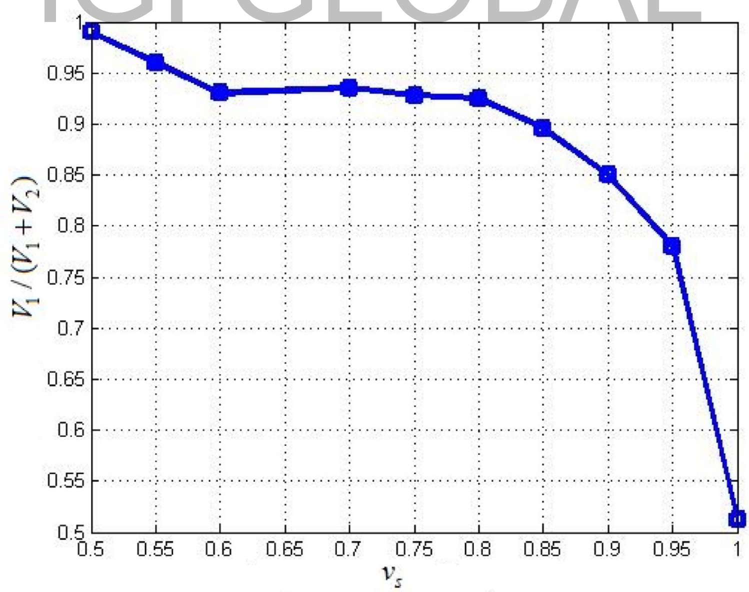


Figure 5 and Figure 6 shows the scenario that one dominant voting pattern ($-1, +1, -1, +1, -1$), i.e., (black, white, black, white, black) appears if we only consider the homogeneous positive social influence, or constantly let $I_{ij} = +1$ in Hopfield network

Figure 5. Group consensus appears under the condition of homogeneous positive influence (the figure illustrates the initial states of group opinions)

Figure 6. Group consensus appears under the condition of homogeneous positive influence (the figure demonstrates the group consensus states, for 100 voters towards 5 options)

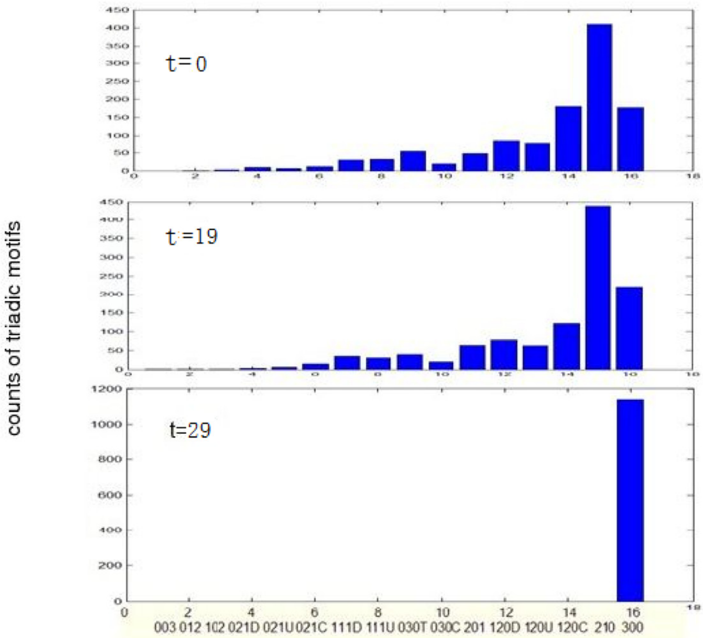
model. This result suggests that collective opinion tends to final consensus state in a homogeneous group-based voting system.

TRIADIC MOTIFS DISTRIBUTION BEFORE AND AFTER BI-POLARIZATION

Furthermore, we investigate the triadic relation motifs distribution before and after bi-polarization by using R package sna (Butts, 2008). We find that the overwhelming structure balance motifs emerge and concurrently with polarization process.

Figure 7 presents the dynamic variation of triadic distributions. At the very beginning ($t = 0$), the upper plot in Figure 7 shows the initial local triads distribution according to randomly generated social influence matrix. We can observe that all 16 types of triads exist in the initial triadic relationships. With the social influence matrix updating, at step $t = 19$ some triad motifs disappear, e.g., Code 003, Code 012 and Code 102, while Code 300, Code 210 become dominant (see middle plot in Figure 7). Finally, at step $t = 29$, other triad motifs

Figure 7. Types of Triad distributions variation vs. social influence updating processes



disappear except balanced triad motif Code 300 (see bottom plot in Figure 7).

As illustrated in Figure 7, with social influence matrix updating, Code 300 becomes dominant. In the end, at step $t = 29$ bi-polarization appears. Corresponding to this global opinion polar state, each triadic relation at the local level also reaches balanced stable state, and only Code 300 remains. This simulation result verifies the internal connection between global pattern and local structure, i.e., opinions polarization in a group is coexisted with local level structure balance.

CONCLUSION

In this paper, we address the implications of three types of social influence based on social identity theory. We investigate the non-positive social impact on group polarization based on Hopfield network model. By simulation we find that bi-polarization pattern tends to emerge with no imposing external intervention, and consensus may occur among group members if the non-positive influence is neglected.

Most literatures suggest that the homogeneous social influence will bring the global stability of social homogeneity, where convergence to one leading polarization is almost irresistible in a closely interconnected or interrelated population. However, in this paper the simulation based on Hopfield network model demonstrates that social homogeneous stable state is highly brittle if "influence ties" are either to be negative or zero.

Compared with Macy et al. (2003), the study in this paper argues that bi-polarization may also be attributed to in-group/out-group differentiation and rejection antagonism, and the difference labeled with the voters' cognition as assumed by social identity theory.

By looking into the identification of group members, the finding indicates that the voting behavior of heterogeneous group is different from that of homogeneous group. The essence of social identity theory, which hold that people maintain an "us" versus "them" portrait during the processes of the collective behaviors, may help explain heterogeneous group opinion polarization.

This conclusion might partially explain a series of recent fifty-fifty voting result in western countries, such as the Bush-Gore 2000 presidential election in US, both the Stoiber-Schroder 2002, and Schroder-Merkel 2005 German elections, and the Prodi-Berlusconi 2006 Italian elections. Galam used the "contrarian effect" to explain these well matched voting phenomenon (Galam, 2007, 2008). In this paper, we illustrate this type of human collective voting pattern from non-positive social influence point of view.

While a person is not only influenced by those who have the same or the opposite social identities, but also (actually even more strongly) influenced by his/her close friends, neighbors etc, such as addressed by Granovetter (1978) and Krackhardt (1996). Next, we may try to include local network structure and various types of spatial effects into our model.

It is worth to point out that our study shows that individuals can self-organize into local structure balance motif after group opinion polarization. This conclusion shows that opinions polarization in a group is coexisted with local level structure balance, which reveals some interesting internal connection between global collective pattern and local social structure stability.

ACKNOWLEDGMENTS

The main ideas of this paper were originally presented at the 6th International Conference on Knowledge, Information and Creativity Support Systems held in Beijing during October 22-24 2011. The conference paper got the best student paper award. This research was supported by National Basic Research Program of China under Grant No. 2010CB731405, National Natural Science Foundation of China under Grant No.71171187 and the Project of Knowledge Innovation Program (PKIP) of Chinese Academy of Sciences under Grant No. KJCX2.YW.W10.

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