Generalized Prioritized Aggregation Operators

Cuiping Wei,[1,*] Xijin Tang[2,†]
[1]*Management School, Qufu Normal University, Rizhao 276826, People's Republic of China*
[2]*Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, People's Republic of China*

This paper deals with multicriteria decision-making problems in which the criteria are partitioned into q categories, and a prioritization relationship exists over categories. We aggregate the criteria in the same priority category by a weighted OWA (ordered weighted averaging) operator and introduce two averaging operators, a generalized prioritized averaging operator and a generalized prioritized OWA operator. In the case with one criterion in each priority category, the two operators reduce to the prioritized averaging operator and the prioritized OWA operator as proposed by Yager.

1. INTRODUCTION

Multicriteria decision-making (MCDM) problems, according to their nature, the policy of the decision maker, and the overall objective of the decision may require the choice of an alternative solution or the ranking of the alternatives from the best to the worst ones based on their satisfactions to a collection of criteria. A central problem in MCDM problems is the aggregation of the satisfactions to the individual criteria to obtain a measure of satisfaction to the overall collection of criteria for each alternative. In general, aggregation methods used reflect the decision maker's imperative and behavior of individual choice.[1,2]

For the case that one associates different importance weights with different criteria, there are several approaches to obtain an overall satisfaction for each alternative to all criteria, such as weighted means and weighted quasiarithmetic means.[3–6] By using these aggregation methods, we allow a compensation between criteria, that is the satisfaction to one criterion can be completely compensated by the satisfaction to another criterion.

*Author to whom all correspondence should be addressed: e-mail: wei_cuiping@yahoo.com.cn.
†e-mail: xjtang@iss.ac.cn.

INTERNATIONAL JOURNAL OF INTELLIGENT SYSTEMS, VOL. 27, 578–589 (2012)

View this article online at wileyonlinelibrary.com. • DOI 10.1002/int.21537

In many real applications, there exists a prioritization relationship over the criteria and we do not want to allow this kind of compensation between criteria. Yager[2,7,8] listed many examples to illustrate this kind of situations, such as selecting a bicycle, an organization decision-making problem, and a document retrieval problem. He suggested that prioritization between criteria can be modeled by making the weights associated with a criterion dependent upon the satisfaction to the higher priority criteria. In Refs. 7 and 8, Yager introduced a prioritized scoring operator for the case that there exist a prioritization between criteria categories and introduced a prioritized averaging operator and an prioritized ordered weighted averaging (OWA) operator for the special case that there is only one element in each criteria category. For other prioritized aggregation techniques, please refer to Refs. 9–11.

For the case that there exists a prioritization between criteria categories, motivated by the work of Yager, we introduce two averaging operators, a generalized prioritized averaging operator and a generalized OWA operator, by using weighted OWA operators[12] to aggregate criteria in the same priority category. For the special case that there is only one criterion in each priority category, the two operators reduce to the prioritized averaging operator and the prioritized OWA operator as proposed by Yager in Refs. 7 and 8.

2. PRELIMINARIES

2.1. A Weighted OWA Operator

MCDM problems, suppose we have a set of criteria $C = \{C_1, C_2, \ldots, C_n\}$ and a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$. We further have a measure of the satisfaction of criteria C_i by each alternative x $(x \in X)$, $C_i(x) \in [0, 1]$. We calculate an overall score $C(x)$ for each alternative x as an aggregation of satisfactions $C_i(x)$:

$$C(x) = F(C_1(x), C_2(x), \ldots, C_n(x)).$$

We then use these overall scores to rank the alternatives.

If the form for F is a weighted averaging (WA) operator f_{wa}, then we calculate

$$C(x) = f_{wa}(C_1(x), C_2(x), \ldots, C_n(x)) = \sum_{i=1}^{n} w_i C_i(x), \tag{1}$$

where w_i are the importance weights associated with the criteria C_i and satisfy $w_i \in [0, 1]$, $\sum_{i=1}^{n} w_i = 1$. In this case, the value $C_i(x)$ of the criterion C_i by the alternative x is weighted according to the weight w_i.

In the case that there is no distinction between the criteria, Yager introduced an OWA operator to aggregate numerical values, which has attracted many researchers to study its properties and applications.[14–16]

Definition 1.[13] *Let* $w = (w_1, w_2, \ldots, w_n)$ *be a weighting vector such that* $w_i \in [0, 1]$, $\sum_{i=1}^{n} w_i = 1$. *A mapping* $f_{owa}^{w}\colon R^n \to R$ *is an OWA operator of dimension* n *if*

$$f_{owa}^{w}(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} w_i b_i, \tag{2}$$

where b_i is the ith largest element in the collection $a_1, a_2, \ldots, a_n$.

From (1) and (2), we can see that the WA operator weights only the value of the jth criterion (or information source) according to the weight w_j, whereas in the OWA operator, each w_j is attached to the jth value in a decreasing order without considering which information source it comes from. To combine the advantages of the two operators, Torra[12] defined a new combination function, called a weighted OWA operator, which considers both the relevance of criteria (as the WA operator) and the relevance of the values (as the OWA operator). In the weighted OWA operator, two weighting vectors, p, corresponding to the relevance of the criteria and w corresponding to the relevance of the values, are considered.

DEFINITION 2.[12] *Let $p = (p_1, p_2, \ldots, p_n)$ and $w = (w_1, w_2, \ldots, w_n)$ be two weighting vectors of dimension n such that*

$$p_i \in [0, 1] \quad and \quad \sum_{i=1}^{n} p_i = 1; \quad w_i \in [0, 1] \quad and \quad \sum_{i=1}^{n} w_i = 1.$$

In this case, a mapping $f_{wowa}^{p,w}$: $R^n \to R$ is a weighted ordered weighted averaging (WOWA) operator of dimension n if

$$f_{wowa}^{p,w}(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} v_i b_i, \tag{3}$$

where b_i is the ith largest element in the collection $a_1, a_2, \ldots, a_n$ and the weight v_i are defined as

$$v_i = w^* \left(\sum_{j=1}^{i} p_{\sigma(j)} \right) - w^* \left(\sum_{j=1}^{i-1} p_{\sigma(j)} \right), \tag{4}$$

with w^* as a monotone increasing function that interpolates the points $(\frac{i}{n}, \sum_{j=1}^{i} w_j)$ together with the point $(0, 0)$ and is required to be a straight line when the points can be interpolated in this way.

From the conditions that the function w^* satisfies in Definition 2, we can get

$$w^*(x) = \sum_{k=1}^{i-1} w_k + w_i(nx - (i-1)) \quad \text{for} \quad \frac{i-1}{n} \leq x \leq \frac{i}{n}. \tag{5}$$

PROPOSITION 1.[12] *The WOWA operator $f_{wowa}^{p,w}$ satisfies the following properties:*

(1) the weight vector $v = (v_1, v_2, \ldots, v_n)$ satisfies $\sum_{i=1}^{n} v_i = 1$.
(2) If p is defined as $p_i = \frac{1}{n}$ for all $i = 1, 2, \ldots, n$, then the WOWA operator $f_{wowa}^{p,w}$ reduces to an OWA operator with a weighting vector w.
(3) If w is defined as $w_i = \frac{1}{n}$ for all $i = 1, 2, \ldots, n$, then the WOWA operator $f_{wowa}^{p,w}$ reduces to a weighted averaging operator with a weighting vector p.
(4) It is an aggregation operator that remains between the minimum and the maximum.
(5) It satisfies idempotency.
(6) It is monotone in relation to the input values.

2.2. Prioritized Aggregation Operators

Yager[7,8] considered criteria aggregation problems in which a prioritization relationship between the criteria exists and proposed a prioritized scoring operator, a prioritized averaging operator and a prioritized OWA operator. The discussed problem is described as follows: Suppose that we have a collection of criteria $C = \{C_1, C_2, \ldots, C_n\}$ and a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$. The collection of criteria C is partitioned into q distinct categories, $H_1, H_2, \ldots, H_q$, such that

$$H_i = \{C_{i1}, C_{i2}, \ldots, C_{in_i}\}.$$

Here C_{ij} are the criteria in category H_i, $C = \bigcup_{i=1}^{q} H_i$, and $n = \sum_{i=1}^{q} n_i$. We assume a prioritization between these categories $H_1 \succ H_2 \succ \cdots \succ H_q$. The criteria in the category H_i have a higher priority than those in H_k if $i < k$. We assume that, for any alternative x in X, we have for each criteria C_{ij} a value $C_{ij}(x) \in [0, 1]$, indicating its satisfaction to criteria C_{ij} . Our aim is to rank the alternatives in X.

Yager[7] introduced a prioritized scoring (PS) operator $f_{ps} : [0, 1]^n \rightarrow [0, 1]$ such that $f_{ps}((a_{11}, a_{12}, \ldots, a_{1n_1}), \ldots, (a_{q1}, a_{q2}, \ldots, a_{qn_q})) = \sum_{i=1}^{q} \left(\sum_{j=1}^{n_i} w_{ij} a_{ij} \right)$. Using this aggregation operator, we can calculate $C(x)$ for alternative x as

$$C(x) = f_{ps}(C_{ij}(x)) = \sum_{i=1}^{q} \left(\sum_{j=1}^{n_i} w_{ij} C_{ij}(x) \right).$$

Here the weights w_{ij} are a function of x and are used to reflect the priority relationship. Yager[7] used the following approach to obtain the weights w_{ij} for a given alternative x: Let $S_0 = 1$, $S_i = \min_j\{C_{ij}(x)\}$, for $i = 1, 2, \ldots, q$, and $T_i = \Pi_{k=1}^{i} S_{k-1}$, for $i = 1, 2, \ldots, q$. Then we take $w_{ij} = T_i$.

With $w_{ij} = T_i$, the aggregation value $C(x)$ for alternative x could be calculated by

$$C(x) = \sum_{i=1}^{q}\sum_{j=1}^{n_i} w_{ij} C_{ij}(x) = \sum_{i=1}^{q} T_i \left(\sum_{j=1}^{n_i} C_{ij}(x)\right). \quad (6)$$

Some alternative methods for calculating S_i were introduced by Yager:[7]

(1) Use the OWA operator to aggregate the priority category $H_i = \{C_{i1}, C_{i2}, \ldots, C_{in}\}$ and suppose

$$S_i = f_{owa}^{W_i}(C_{i1}(x), C_{i2}(x), \ldots, C_{in_i}(x)) = \sum_{k=1}^{n_i} W_{ik} b_{ik}(x), \quad (7)$$

where W_i is the OWA weighting vector associating with each priority category H_i and $b_{ik}(x)$ is the kth largest of $C_{ij}(x)$. The components W_{ik} of W_i are such as $W_{ik} \in [0, 1]$ and $\sum_{k=1}^{n_i} W_{ik} = 1$.

(2) Suppose that there is an additional local weight associating with each criterion in H_i and the form for H_i is

$$H_i = \{(C_{ij}, g_{ij}) \mid j = 1, 2, \ldots, n_i\},$$

where g_{ij} indicates the importance of C_{ij}, satisfying $g_{ij} \in [0, 1]$ and $\sum_{j=1}^{n_i} g_{ij} = 1$. In this case, S_i is calculated by

$$S_i = f_{wa}(C_{i1}(x), C_{i2}(x), \ldots, C_{in_i}(x)) = \sum_{j=1}^{n_i} g_{ij} C_{ij}(x). \quad (8)$$

Another method for calculating S_i involves the idea of combing these local weights with the OWA weights. Suppose $W_i = (W_{i1}, W_{i2}, \ldots, W_{in_i})$ is the OWA weighting vector associating with each priority category H_i, $b_{ik}(x)$ is the kth largest value of $C_{ij}(x)$, and d_{ik} is the importance weight associated with the kth largest value of $C_{ij}(x)$. We calculate

$$h_{ik} = \frac{d_{ik} W_{ik}}{\sum_{k=1}^{n_i} d_{ik} W_{ik}}.$$

Using this, we calculate

$$S_i = \sum_{k=1}^{n_i} h_{ik} b_{ik}(x). \quad (9)$$

For the case that there exists prioritization relationship between categories, Yager used different methods to calculate S_i, but the above aggregation operator defined by (6) is a scoring operator and not an averaging operator, which is illustrated by Yager in Ref.[7] Furthermore, in the case with one criterion in each category H_i, Yager[7,8] proposed a prioritized averaging (PA) and a prioritized OWA (POWA) operator.

In the following section, we introduce two averaging operators for the general case that a prioritization exists between categories H_i.

3. GENERALIZED PRIORITIZED AVERAGING OPERATORS

Here we also assume that we have a collection of criteria C partitioned into q distinct categories, $H_1, H_2, \ldots, H_q$, such that $H_i = \{C_{i1}, C_{i2}, \ldots, C_{in_i}\}$. Here $C = \bigcup_{i=1}^{q} H_i$ and $n = \sum_{i=1}^{q} n_i$. We assume a prioritization between these categories $H_1 \succ H_2 \succ \cdots \succ H_q$. For each H_i, we assume we have

$$H_i = \{(C_{ij}, g_{ij}), j = 1, 2, \ldots, n_i\},$$

where g_{ij} are the additional local weights associating criteria C_{ij} in H_i and satisfy $g_{ij} \in [0, 1]$ and $\sum_{j=1}^{n_i} g_{ij} = 1$. $C_{ij}(x)$ are defined as in Section 2.2.

To aggregate the criteria in each category, we associate with each category H_i an OWA weighting vector W_i of dimension n_i. The components W_{ik} of W_i satisfy $W_{ik} \in [0, 1]$ and $\sum_{k=1}^{n_i} W_{ik} = 1$. Since $g_i = (g_{i1}, g_{i2}, \ldots, g_{in_i})$ be the local weighting vector of the priority category H_i and W_i be the OWA weighting vector associated with H_i, we use the WOWA operator to aggregate the criteria in each category and take S_i as the WOWA-aggregated value for the collection $C_{i1}(x), C_{i2}(x), \ldots, C_{in_i}(x)$. That is,

$$S_i = f_{wowa}^{g_i, W_i}(C_{i1}(x), C_{i2}(x), \ldots, C_{in_i}(x)). \tag{10}$$

We suppose $S_0 = 1$, $T_i = \Pi_{k=1}^{i} S_{k-1} = S_{i-1} T_{i-1}$ for $i = 1, 2 \ldots, q$, and $T = \sum_{i=1}^{q} T_i$. Then we can obtain normalized weights $t_i = \frac{T_i}{T}$ associated with category H_i $(i = 1, 2, \ldots q)$. Using this we calculate, the aggregated value

$$C(x) = F(C_{ij}(x)) = \sum_{i=1}^{q} t_i S_i. \tag{11}$$

We refer to it as the generalized prioritized averaging (GPA) operator.

Remark 1. (1) If $n_i = 1$ for all i, then $S_i = C_i(x)$ and the GPA operator reduces to the PA operator proposed by Yager in Ref. 7.

(2) If $g_{ij} = \frac{1}{n_i}$ for all i and j, then

$$S_i = f_{owa}^{W_i}(C_{i1}(x), C_{i2}(x), \ldots, C_{in_i}(x)).$$

(3) If $W_{ij} = \frac{1}{n_i}$ for all i and j , then

$$S_i = f_{wa}^{g_i}(C_{i1}(x), C_{i2}(x), \ldots, C_{in_i}(x)).$$

For the cases (2) and (3), the formulas for calculating S_i are the same as the formulas (7) and (8) used by Yager to aggregate the priority category H_i in the prioritized scoring operator.

PROPOSITION 2. *The aggregation* $F : [0, 1]^n \to [0, 1]$ *defined by Formula (11) satisfies the following properties:*

(1) It is an aggregation operator that remains between the minimum and the maximum.
(2) It satisfies idempotency.
(3) It is monotone in relation to the value $C_{ij}(x)$.

Proof. (1) From the formula (10) for calculating S_i and (4) in Proposition 1, we can obtain that

$$\min_j\{C_{ij}(x)\}\} \le S_i \le \max_j\{C_{ij}(x)\}.$$

Since $\min_{ij}\{C_{ij}(x)\}\} \le \min_j\{C_{ij}(x)\}\}$ and $\max_{ij}\{C_{ij}(x)\}\} \ge \max_j\{C_{ij}(x)\}\}$, we have

$$\min_{ij}\{C_{ij}(x)\}\} \le S_i \le \max_{ij}\{C_{ij}(x)\}.$$

Also since $\sum_{i=1}^q t_i = 1$, $C(x) = \sum_{i=1}^q t_i S_i$ remains between the minimum and the maximum.

(2) Since the WOWA operator satisfies idempotency, we have $S_i = a$ if $C_{ij}(x) = a$ for all i and j. Also since $\sum_{i=1}^q t_i = 1$, we have $C(x) = \sum_{i=1}^q t_i S_i = a$.

(3) For

$$C(x) = F((C_{11}(x), C_{12}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), C_{q2}(x), \ldots, C_{qn_q}(x)))$$
$$= \sum_{i=1}^{q} \frac{T_i}{T} S_i,$$

obviously, $F(C_{ij}(x)) = 1 + \frac{S_1 S_2 \ldots S_q - 1}{T}$. To obtain the monotonicity, we have to show that $\frac{\partial F}{\partial C_{ij}(x)} \ge 0$ for each $i = 1, 2, \ldots, q$, $j = 1, 2, \ldots, n_i$. Using the derivation rule of composite functions, we have

$$\frac{\partial F}{\partial C_{ij}(x)} = \frac{\partial F}{\partial S_i}\frac{\partial S_i}{\partial C_{ij}(x)}.$$

By Proposition 1, one can obtain that each S_i is increasing with respect to $C_{i1}(x)$, $C_{i2}(x)$, $\ldots$, $C_{in_i}(x)$, which implies that $\frac{\partial S_i}{\partial C_{ij}(x)} \geq 0$ for each $i = 1, 2, \ldots, q$, $j = 1, 2, \ldots, n_i$. Next we prove $\frac{\partial F}{\partial S_i} \geq 0$ for each $i = 1, 2, \ldots, q$.

Obviously,

$$\frac{\partial F}{\partial S_q} = \frac{S_1 S_2 \cdots S_{q-1}}{T} \geq 0,$$

$$\frac{\partial F}{\partial S_{q-1}} = \frac{S_1 S_2 \cdots S_{q-2} S_q T - (S_1 S_2 \cdots S_q - 1) S_1 S_2 \cdots S_{q-2}}{T^2} \geq 0.$$

For $1 \leq i \leq q - 2$, we have

$$\frac{\partial F}{\partial S_i} = \frac{S_1 S_2 \cdots S_{i-1} S_{i+1} \cdots S_q T - (S_1 S_2 \cdots S_q - 1)\frac{\partial T}{\partial S_i}}{T^2}.$$

Note that

$$\begin{aligned}\frac{\partial T}{\partial S_i} &= S_1 S_2 \cdots S_{i-1}(1 + S_{i+1} + S_{i+1}S_{i+2} + \ldots + S_{i+1}S_{i+2}\cdots S_{q-1}) \\ &= T_i(1 + S_{i+1} + \cdots + S_{i+1}\cdots S_{q-1}) \geq 0.\end{aligned}$$

We decompose T as

$$T = (T_1 + T_2 + \cdots + T_i) + T_{i+1}(1 + S_{i+1} + S_{i+1}S_{i+2} + \cdots + S_{i+1}\cdots S_{q-1}).$$

Hence

$$\begin{aligned}&S_1 S_2 \cdots S_{i-1} S_{i+1} \cdots S_q T - (S_1 S_2 \cdots S_q - 1)\frac{\partial T}{\partial S_i} \\ &= S_1 S_2 \cdots S_{i-1} S_{i+1} \cdots S_q (T_1 + T_2 + \cdots + T_i) \\ &\quad + S_1 S_2 \cdots S_{i-1} S_{i+1} \cdots S_q T_{i+1}(1 + S_{i+1} + \cdots + S_{i+1}\cdots S_{q-1}) \\ &\quad - S_1 S_2 \cdots S_q T_i (1 + S_{i+1} + S_{i+1}S_{i+2} + \cdots + S_{i+1}\cdots S_{q-1}) + \frac{\partial T}{\partial S_i} \\ &= S_1 S_2 \cdots S_{i-1} S_{i+1} \cdots S_q (T_1 + T_2 + \cdots + T_i) + \frac{\partial T}{\partial S_i} \geq 0,\end{aligned}$$

which yields that $\frac{\partial F}{\partial S_i} \geq 0$, for $1 \leq i \leq q - 2$. ■

Example C. onsider the following prioritized collection of criteria:
$H_1 = \{(C_{11}, 0.6), (C_{12}, 0.4)\}$,
$H_2 = \{(C_{21}, 1)\}$,
$H_3 = \{(C_{31}, 0.2), (C_{32}, 0.4), (C_{33}, 0.4)\}$,
$H_4 = \{(C_{41}, 0.8), (C_{42}, 0.2)\}$.
Assume for alternative x, we have
$C_{11}(x) = 0.7$,
$C_{12}(x) = 1$, $C_{21}(x) = 0.9$,
$C_{31}(x) = 0.8$,
$C_{32}(x) = 1$, $C_{33}(x) = 0.2$,
$C_{41}(x) = 1$, $C_{42}(x) = 0.9$.
We associate with each priority category H_i an OWA weighting vector W_i as follows:
$W_1 = (0.3, 0.7)$,
$W_2 = (1)$,
$W_3 = (0.3, 0.5, 0.2)$,
$W_4 = (0.5, 0.5)$.

For priority category H_1, using W_1 and Formula (5), we can get the function w^* such that

$$w^*(x) = \begin{cases} 0.6x, & 0 \leq x \leq 0.5; \\ 0.3 + 0.7(2x - 1), & 0.5 \leq x \leq 1. \end{cases}$$

In this example $C_{12}(x) = 1 > C_{11}(x) = 0.7$. From this, we get
$w^*(g_{12}) = 0.24$, $w^*(g_{12} + g_{11}) = 1$.
Using this and Formula (4), we get the WOWA weighting vector $V_i = (v_{11}, v_{12})$, where
$v_{11} = 0.24 - 0 = 0.24$, $v_{12} = 1 - 0.24 = 0.76$.
So we get the aggregated value S_1 for category H_1:
$S_1 = f_{wowa}^{g_1, W_1}(C_{11}(x), C_{12}(x)) = 0.24 \times 1 + 0.76 \times 0.7 = 0.772$.
Similarly, we calculate
$S_2 = f_{wowa}^{g_2, W_2}(C_{21}(x)) = 0.9$,
$S_3 = f_{wowa}^{g_3, W_3}(C_{31}(x), C_{32}(x), C_{33}(x)) = 0.7$,
$S_4 = f_{wowa}^{g_4, W_4}(C_{41}(x), C_{42}(x)) = 0.98$.
Using this, we get
$T_1 = 1$,
$T_2 = S_1 T_1 = 0.772$,
$T_3 = S_2 T_2 = 0.695$,
$T_4 = S_3 T_3 = 0.487$,
and
$T = \sum_{i=1}^{4} T_i = 2.954$.
Thus, we obtain
$t_1 = \frac{T_1}{T} = 0.34$,
$t_2 = \frac{T_2}{T} = 0.26$,

$t_3 = \frac{T_3}{T} = 0.24$,
$t_4 = \frac{T_4}{T} = 0.16$.
We now calculate , $C(x) = \sum_{i=1}^{4} t_i S_i = 0.82$.

In the preceding discussion, we use the WOWA operator to aggregate criteria in each category H_i and obtain the aggregated value, denoted by S_i, which indicates the overall satisfaction to the category H_i by alternative x. We then aggregate the collection $S_1, S_2, \ldots, S_q$ by using the PA operator and obtain the multicriteria aggregation for alternative x.

We now consider to aggregate the collection $S_1, S_2, \ldots, S_q$ based on the WOWA operator. We introduce an aggregation operator $F : [0, 1]^n \to [0, 1]$ such that

$$F((a_{11}, a_{12}, \ldots, a_{1n_1}), \ldots, (a_{q1}, q_{q2}, \ldots, q_{qn_q})) = f_{wowa}^{t,W}(S_1, S_2, \ldots, S_q). \quad (12)$$

where $S_i = f_{wowa}^{g_i, W_i}(a_{i1}, a_{i2}, \ldots, a_{in_i})$, $W = (w_1, w_2, \ldots, w_q)$ is the OWA weighting vector associated with the categories, and t is the weighting vector determined by the prioritization between categories and the aggregated values S_i. We refer to it as the generalized prioritized (GPOWA) operator.

Using this aggregation operator, we calculate $C(x)$ for any alternative x as

$$C(x) = F(C_{ij}(x)) = f_{powa}^{t,W}(S_1, S_2, \ldots, S_q),$$

where $S_i = f_{wowa}^{g_i, W_i}(C_{i1}(x), C_{i2}(x), \ldots, C_{in_i}(x))$ and W is the OWA weighting vector associated with the criteria categories. The component t_i of the weighting vector t is calculated by $t_i = \frac{T_i}{T},\ i = 1, 2, \ldots, q$. Here $S_0 = 1$, $T_i = \Pi_{k=1}^{i} S_{k-1} = S_{i-1} T_{i-1}$ for $i = 1\ to\ q$, and $T = \sum_{i=1}^{q} T_i$.

Remark 2. (1) In the case that there is only one element in each priority category, the GPOWA operator reduces to the POWA operator proposed by Yager in Ref. 8.

(2) If W is defined as $w_i = \frac{1}{n}$ for all $i = 1, 2, \ldots, q$, the GPOWA operator reduces to the GPA operator.

(3) In the GPA and the GPOWA operators, we can adopt the Formula (9) to aggregate the criteria in each category. That is, we calculate

$$S_i = \sum_{k=1}^{n_i} h_{ik} b_{ik}(x).$$

Here $h_{ik} = \frac{d_{ik} W_{ik}}{\sum_{k=1}^{n_i} d_{ik} W_{ik}}$, $b_{ik}(x)$ is the kth largest value of $C_{ij}(x)$ $(j = 1, 2, \ldots, n_i)$, and d_{ik} is the importance weight associated with the kth largest value of $C_{ij}(x)$.

4. CONCLUSIONS

We consider criteria aggregation problems in which a prioritization relationship exists over the criteria. We use the WOWA operator to aggregate criteria in each category H_i and obtain the aggregated value, denoted by S_i, which indicates the overall satisfaction to the category H_i by alternative x. We then determie the importance weights of the categories by using the collection of $S_1, S_2, \ldots, S_q$ and the priority relationship between the categories. With these importance weights and the WA or WOWA operator, we aggregate the collection $S_1, S_2, \ldots, S_q$ and obtain the multicriteria aggregation. On the basis of these ideas, we introduce two averaging aggregation operators, which generalize the PA and POWA operators introduced by Yager in Refs. 7,8.

Acknowledgments

The work was partly supported by the National Natural Science Foundation of China (71171187, 11071142), the National Basic Research Program of China (2010CB731405), and the Ministry of Education Foundation of Humanities and Social Sciences (10YJC630269).

References

1. Luo X, Jennings NR. A spectrum of compromise aggregation operators for multi-attribute decision making. Artif Intell 2007;171(2–3):161–184.
2. Yager RR. Modeling prioritized multi-criteria decision making. IEEE Trans Syst Man Cybernet, Part B: Cybernet 2004;34(6):2396–2404.
3. Yager RR. Multiple objective decision making using fuzzy sets. Int J Man-Machine Stud 1977;9:375–382.
4. Yager RR. A note on weighted queries in information retrieval systems J Am Soc Inform Sci 1987;38:223–224.
5. Calvo T, Kolesarova A, Komornikova A, Mesiar R. Aggregation operators: properties, classes and construction methods. In: Calvo T, Mayor G, Mesiar R, editors. Aggregation operators: New trends and applications. Studies in Fuzziness and Soft Computing, Vol 97. Heidelberg, Germany: Physica-Verlag; 2002. pp 3–104.
6. Calvo T, Mesiar R. Weighted means based on triangular conorms. Int J Uncertain Fuzziness Knowl Based Syst 2001;9;183–196.
7. Yager RR. Prioritized aggregation operators. Int J Approx Reason 2008;48:263–274.
8. Yager RR. Prioritized OWA aggregation. Fuzzy Optim Decis Making 2009;8(3):245–262.
9. Yan HB, Huynh VN, Nakamori Y, Murai T. On prioritized weighted aggregation in multi-criteria decision making. Expert Syst Appl 2011:38:812–823.
10. Chen SJ, Chen SM. A prioritized information fusion method for handling fuzzy decision-making problems. Appl Intell 2005;22:219–232.
11. Chen SJ, Chen SM. A prioritized information fusion algorithm for handling multi-criteria fuzzy decision-making problems. In: Proc 1st Int Conf on Fuzzy Systems and Knowledge Discovery: Computational Intelligence for the E-Age (FSDK'02), Orchid Country Club, Singapore; November 18–22, 2002. Vol 2, pp 413–417.
12. Torra V. The weighted OWA operator. Int J Intell Syst 1997;12:153–166.
13. Yager RR. On ordered weighted averaging operators in multi-criteria decision making. IEEE Trans Syst Man Cybernet 1988;18:183–190.

14. Xu ZS. An overview of methods for determing OWA weights. Int J Intell Syst 2005;20843–865.
15. O'Hagan M. A Fuzzy Neuron Based on Maximum Entropy Ordered Weighted Averaging. In: Proc 24th Annual IEEE Asilomar Conf on Signals, Systems and Computers, Pacific Grove, CA. November 5–7, 1990. pp 618–623.
16. Fuller R, Majlender P. On obtaining minimal variability OWA weights. Fuzzy Sets Syst 2003;136:203–215.