A New Method for Ranking Intuitionistic Fuzzy Numbers

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ABSTRACT

In this paper the ranking method for intuitionistic fuzzy numbers is studied. The authors first define a possibility degree formula to compare two intuitionistic fuzzy numbers. In comparison with Chen and Tan's score function, the possibility degree formula provides additional information for the comparison of two intuitionistic fuzzy numbers. Based on the possibility degree formula, the authors give a possibility degree method to rank n intuitionistic fuzzy numbers, which is used to rank the alternatives in multi-criteria decision making problems.

Keywords: *Intuitionistic Fuzzy Number, Multi-Criteria Decision Making, Possibility Degree Method, Ranking, Score Function*

1. INTRODUCTION

Since Zadeh (1965) introduced fuzzy sets theory, some generalized forms have been proposed to deal with imprecision and uncertainty. Atanassov (1986) introduced the concept of an intuitionistic fuzzy set (IFS) characterized by a membership function and a non-membership function. Gau and Buehrer (1993) introduced the concept of vague sets. Bustince and Burillo (1996) showed that vague sets are IFSs. IFSs have been found to be more useful to deal with vagueness and uncertainty problems than fuzzy sets, and have been applied to many different fields.

For the fuzzy multiple criteria decision making (MCDM) problems, the degree of satisfiability and non-satisfiability of each alternative with respect to a set of criteria is often represented by an intuitionistic fuzzy number (IFN), which is an element of an IFS (Liu, 2003; Xu, 2007). The comparison between alternatives is equivalent to the comparison of IFNs. Chen and Tan (1994) provided a score function to compare IFNs. Hong and Choi (2000) pointed out the defects and proposed an improved technique based on the score function and accuracy function. Later, Li (2001) and Liu (2003) gave a series of improved score functions. The above functions are called evaluation functions. By using these evaluation functions, we can obtain certain rank of the IFNs. Since IFNs are of fuzziness, the comparison between them may also be expected to reflect the uncertainty of ranking objectively.

In this paper, by extending the possibility degree formula of interval values (Wang, Yang,

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& Xu, 2005; Xu & Da, 2003) to IFNs, we propose a possibility degree method for ranking n IFNs. And the ranking result by the proposed method may reflect the uncertainty of IFSs, and then provide more information to decision makers.

2. POSSIBILITY DEGREE METHOD FOR RANKING INTUITIONISTIC FUZZY NUMBERS

2.1. Possibility Degree Formula for Ranking Two Intuitionistic Fuzzy Numbers and Its Properties

Let $I=[0,1]$, $\vee=\max$, $\wedge=\min$.

Definition 2.1. (Atanassov, 1986) Let X be an ordinary finite non-empty set. An intuitionistic fuzzy set on X is an expression given by:
$A=\{\langle x,u_A(x),v_A(x)\rangle \mid x\in X\}$, where $u_A: X\to I$, $v_A: X\to I$, with the condition $u_A(x)+v_A(x)\le 1$ for all x in X; $u_A(x)$ and $v_A(x)$ denote, respectively, the membership degree and the non-membership degree of the element x in A. We abbreviate "intuitionistic fuzzy set" to IFS and represent IFS(X) the set of all the IFS on X. We call $\pi_A(x)=1-u_A(x)-v_A(x)$ the degree of hesitation (or uncertainty) associated with the membership of element x in A.

According to Liu (2003) and Xu (2007), for an IFS $A=\{(u_A(x),v_A(x))\mid x\in x\}$, the pairs $(u_A(x),v_A(x))$ is called an intuitionistic fuzzy number (IFN). For convenience we denote an IFN by (a,b), where $a\in I$, $b\in I$, $a+b\le 1$. Let Q be the set of all the IFNs.

Definition 2.2. (Xu, 2007) Let:
$\alpha_i=(a_i,b_i)\in Q$, $i=1,2$, then

1) $(a_1,b_1)=(a_2,b_2)\Leftrightarrow a_1=a_2, b_1=b_2$;
2) $(a_1,b_1)\ge(a_2,b_2)\Leftrightarrow a_1\ge a_2, b_1\le b_2$;
3) $(a_1,b_1)>(a_2,b_2)\Leftrightarrow a_1 \ge a_2, b_1\le b_2 \,\&\, (a_1,b_1)\ne(a_2,b_2)$
4) $\alpha_1=(b_1,a_1)$;
5) $\alpha_1+\alpha_2=(a_1+a_2-a_1a_2, b_1b_2)$;
6) $\alpha_1\alpha_2=(a_1a_2, b_1+b_2-b_1b_2)$;
7) $\lambda\alpha_1=\left(1-(1-a_1)^\lambda, b_1^\lambda\right), \lambda>0$;
8) $\alpha_1^\lambda=\left(a_1^\lambda, 1-(1-b_1)^\lambda\right), \lambda>0$.

For the practical MCDM problems, experts need to obtain the rank of the alternatives. Suppose the comprehensive evaluation value of each alternative is represented by an IFN α, where $\alpha=(a,b)$, which indicates the degree of satisfiability and non-satisfiability of each alternative with respect to all the attributes. The larger the degree of hesitation $\pi(\alpha)$, which is equal to $1-a-b$, the bigger the possible change of the degree of satisfiability and non-satisfiability of the alternative for the experts. As the comprehensive evaluation value is denoted by (a,b), the degree of satisfiability of the alternative for the experts is actually an interval value written as $[a,a+\pi(\alpha)]$. Similarly, the degree of non-satisfiability of the alternative for the experts can be written as $[b,b+\pi(\alpha)]$. Therefore, the comparison between IFNs can be solved by using the possibility degree formula of interval values. Next we extend the possibility degree method of interval-valued numbers (Wang, Yang, & Xu, 2005; Xu

& Da, 2003) to intuitionistic fuzzy sets and define a possibility degree formula to compare two IFNs.

Definition 2.3. Let $\alpha_1 = (a_1, b_1)$, $\alpha_2 = (a_2, b_2)$, $\pi(\alpha_1) = 1 - a_1 - b_1$, $\pi(\alpha_2) = 1 - a_2 - b_2$. If $\pi(\alpha_1) = \pi(\alpha_2) = 0$, we call:

$$p(\alpha_1 > \alpha_2) = \begin{cases} 1, & a_1 > a_2, \\ 0, & a_1 < a_2, \\ 1/2, & a_1 = a_2, \end{cases}$$

the possibility degree of $\alpha_1 > \alpha_2$.

Definition 2.4. Let $\alpha_1 = (a_1, b_1), \alpha_2 = (a_2, b_2)$, $\pi(\alpha_1) = 1 - a_1 - b_1, \pi(\alpha_2) = 1 - a_2 - b_2$. If $\pi(\alpha_1)$ and $\pi(\alpha_2)$ are not zero simultaneously, we call:

$$p(\alpha_1 > \alpha_2) = \frac{\max\{0, (a_1 + \pi(\alpha_1)) - a_2\} - \max\{0, a_1 - (a_2 + \pi(\alpha_2))\}}{\pi(\alpha_1) + \pi(\alpha_2)}$$

the possibility degree of $\alpha_1 > \alpha_2$.

Definition 2.5. If $p(\alpha_1 > \alpha_2) > p(\alpha_2 > \alpha_1)$, then α_1 is superior to α_2 with the degree of $p(\alpha_1 > \alpha_2)$, denoted by $\alpha_1 \overset{p(\alpha_1 > \alpha_2)}{\succ} \alpha_2$;

If $p(\alpha_1 > \alpha_2) = p(\alpha_2 > \alpha_1) = 0.5$, then α_1 is indifferent with α_2, denoted as $\alpha_1 \overset{0.5}{\sim} \alpha_2$;

If $p(\alpha_2 > \alpha_1) > p(\alpha_1 > \alpha_2)$, then α_1 is inferior to α_2 with the degree of $p(\alpha_2 > \alpha_1)$, denoted as $\alpha_1 \overset{p(\alpha_2 > \alpha_1)}{\prec} \alpha_2$.

It is easy to prove that:

Theorem 2.1. Let $\alpha_1 = (a_1, b_1)$, $\alpha_2 = (a_2, b_2)$, then:

(1) $0 \le p(\alpha_1 > \alpha_2) \le 1$;

(2) $p(\alpha_1 > \alpha_2) = 1 \Leftrightarrow a_1 \ge a_2 + \pi(\alpha_2)$;

(3) $p(\alpha_1 > \alpha_2) = 0 \Leftrightarrow a_2 \ge a_1 + \pi(\alpha_1)$;

(4) (complementarity) $p(\alpha_1 > \alpha_2) + p(\alpha_2 > \alpha_1) = 1$; especially if $\alpha_1 = \alpha_2$, then $p(\alpha_1 > \alpha_2) = p(\alpha_2 > \alpha_1) = \frac{1}{2}$;

(5) If $a_1 \le a_2, b_1 \le b_2$, then $p(\alpha_1 > \alpha_2) \ge \frac{1}{2}$ if and only if $a_1 - b_1 \ge a_2 - b_2$; furthermore, $p(\alpha_1 > \alpha_2) = 0.5$ if and only if $a_1 - b_1 = a_2 - b_2$;

(6) If $a_1 \ge a_2, b_1 \le b_2$, then $p(\alpha_1 > \alpha_2) \ge 0.5$;

(7) (transitivity) For $\alpha_1 = (a_1, b_1)$, $\alpha_2 = (a_2, b_2)$, $\alpha_3 = (a_3, b_3)$, if $p(\alpha_1 > \alpha_2) > 1/2$ and $p(\alpha_2 > \alpha_3) \ge 1/2$ or $p(\alpha_1 > \alpha_2) \ge 1/2$ and $p(\alpha_2 > \alpha_3) > 1/2$, then $p(\alpha_1 > \alpha_3) > 1/2$; if $p(\alpha_1 > \alpha_2) = 1/2$ and $p(\alpha_2 > \alpha_3) = 1/2$, then $p(\alpha_1 > \alpha_3) = 1/2$.

For the IFN $\alpha = (a, b)$, Chen and Tan (1994) defined the score function $S(\alpha) = a - b$ and used it to compare two IFNs. Therorem 2.2 shows the possibility degree formula can achieve the same ranking as the score function.

Theorem 2.2. For any two IFNs $\alpha_1 = (a_1, b_1)$ and $\alpha_2 = (a_2, b_2)$, $p(\alpha_1 > \alpha_2) \geq 1/2$ if and only if $S(\alpha_1) \geq S(\alpha_2)$; furthermore, $p(\alpha_1 > \alpha_2) = 1/2$ iff $S(\alpha_1) = S(\alpha_2)$.

For the proof, please refer to Wei and Tang (2010).

2.2. Possibility Degree Method for Ranking n Intuitionistic Fuzzy Numbers

Here we introduce the possibility degree method for ranking n intuitionistic fuzzy numbers $\alpha_1, \alpha_2, \cdots, \alpha_n$.

Step 1. By using pairwise comparisons among n intuitionistic fuzzy numbers $\alpha_1, \alpha_2, \cdots, \alpha_n$, we construct a possibility degree matrix P:

$$P = \begin{pmatrix} 1/2 & p_{12} & \cdots & p_{1n} \\ p_{21} & 1/2 & \cdots & p_{2n} \\ \vdots & \vdots & & \vdots \\ p_{n1} & p_{n2} & \cdots & 1/2 \end{pmatrix},$$

where $p_{ij} =$

$$p(\alpha_1 > \alpha_2) = \frac{\max\{0, (a_1 + \pi(\alpha_1)) - a_2\} - \max\{0, a_1 - (a_2 + \pi(\alpha_2))\}}{\pi(\alpha_1) + \pi(\alpha_2)}.$$

Step 2. Construct the preference relation matrix M from the possibility degree matrix P:

$$M = \begin{pmatrix} 1 & m_{12} & \cdots & m_{1m} \\ m_{21} & 1 & \cdots & m_{2m} \\ \vdots & \vdots & & \vdots \\ m_{m1} & m_{m2} & \cdots & 1 \end{pmatrix}$$

where for any $i \neq j$,

$$m_{ij} = \begin{cases} 1, & p_{ij} \geq 0.5, \\ 0, & p_{ij} < 0.5. \end{cases}$$

Step 3. Find out the rows in which the elements are all equal to 1 in M. We mark the labels of these rows as $J = \{j_1, j_2, \cdots, j_s\}$. From the transitivity of the possibility degree matrix M, we can easily obtain that the corresponding compared IFSs $\alpha_{j_1}, \alpha_{j_2}, \cdots, \alpha_{j_s}$ are indifferent. Let $X_1 = \{\alpha_{j_1}, \alpha_{j_2}, \cdots \alpha_{j_s}\}$. Remove the elements in rows $j_1, \cdots, j_s$ and columns $j_1, \cdots, j_s$ from the matrix M, and the remained elements construct the matrix M_1. Then find out the rows in which the elements are all equal to 1 in M_1, and denotes X_2 to the set of corresponding IFNs, which are also indifferent. Repeat the operation, we can divide the set of n intuitionistic fuzzy numbers into $X_1, X_2, \cdots, X_l$.

Step 4. If X_i just has one element α_{k_i}, then the rank of IFNs fuzzy numbers $\alpha_1, \alpha_2, \cdots, \alpha_n$ is

$$\alpha_{k_1} \overset{p(\alpha_{k_1} > \alpha_{k_2})}{\succ} \alpha_{k_2} \overset{p(\alpha_{k_2} > \alpha_{k_3})}{\succ} \cdots \overset{p(\alpha_{k_{n-2}} > \alpha_{k_{n-1}})}{\succ} \alpha_{k_{n-1}} \overset{p(\alpha_{k_{n-1}} > \alpha_{k_n})}{\succ} \alpha_{k_n}$$

; if there are several IFNs in X_i, we may let $X_i = \{\alpha_k, \alpha_m, \alpha_l\}$ for example, then calculate the average possibility degree which α_i is superior to the other intuitionistic fuzzy numbers by the formula:

$$w_i = \frac{1}{n-1} \sum_{j=1, j \neq i}^{n} p_{ij}, i = k, l, m$$

to rank $\alpha_k, \alpha_m, \alpha_l$. For any $i, j = k, l, m$, if $w_i > w_j$, then α_i is quasi-superior to α_j, denoted as $\alpha_i \rhd \alpha_j$. If $w_i = w_j$, we say α_1 is quasi- indifferent with α_2, denoted as $\alpha_i \approx \alpha_j$.

Remark The above method for ranking n intuitionistic fuzzy numbers is different from the method for ranking interval values given by Wang, Yang, and Xu (2005), since we consider the case that there exist some elements that are equal to 0.5 in possibility degree matrix P. It is also different from the method by Xu & Da (2003) due to different mechanism.

Example 2.1. Given 4 IFNs: $\alpha_1 = (0.1, 0.6)$, $\alpha_2 = (0.3, 0.4)$, $\alpha_3 = (0.2, 0.6)$, $\alpha_4 = (0.1, 0.5)$. Now we use the possibility degree method to rank the 4 IFNs.

By Step 1 and Step 2, we obtain the possibility degree matrix P and the preference relation matrix M:

$$P = \begin{pmatrix} 1 & 0.1667 & 0.4 & 0.4286 \\ 0.8333 & 1 & 0.8 & 0.7143 \\ 0.6 & 0.2 & 1 & 0.5 \\ 0.5714 & 0.2857 & 0.5 & 1 \end{pmatrix},$$

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}.$$

By Step 3, we get $X_1 = \{\alpha_2\}$, $X_2 = \{\alpha_4, \alpha_3\}$ and $X_3 = \{\alpha_1\}$.

Since

$\frac{1}{3}(0.6 + 0.2 + 0.5) < \frac{1}{3}(0.5714 + 0.2857 + 0.5)$,

we get α_4 is quasi-superior to α_3 by **Step 4**. Thus we get the ranking result of 4 IFNs, $\alpha_2 \overset{0.7143}{\succ} \alpha_4 \rhd \alpha_3 \overset{0.6}{\succ} \alpha_1$.

Remark. Since $p(\alpha_4 > \alpha_3) = 1/2$ for IFNs α_3 and α_4, we have α_3 and α_4 are indifferent, i.e. $\alpha_4 \overset{0.5}{\sim} \alpha_3$, which only describes the comparison information between α_4 and α_3, and still cannot distinguish the two IFNs. Thus we make further comparison using Step 4 to combine the comparison information of either α_4 or α_3 with other IFNs in the possibility degree matrix P, and get the result that α_4 is quasi-superior to α_3.

If we adopt the score function $S(\alpha) = a - b$ to rank $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, we get $S(\alpha_1) = -0.5$, $S(\alpha_2) = -0.1$, $S(\alpha_3) = -0.4$, $S(\alpha_4) = -0.4$. The ranking of the 4 IFNs is $\alpha_2 \succ \alpha_4 \sim \alpha_3 \succ \alpha_1$. It can be seen that by our method, we obtain α_2 is superior to α_4 with the amount of 0.7143 in the possibility degree and α_3 is superior to α_1 with the amount of 0.6, besides the same ranking order as that given by score function. Obviously our method brings more information than the ranking by the score function.

3. A DECISION-MAKING METHOD BASED ON INTUITIONISTIC FUZZY INFORMATION

For a MCDM problem, let $X = \{x_1, x_2, \cdots, x_m\}$ be a set of options, $C = \{c_1, c_2, \cdots, c_n\}$ be a set of criteria and $D = (\alpha_{ij})_{m\times n} = ((a_{ij}, b_{ij}))_{m\times n}$ be a decision making matrix, where the degree of satisfiability and non-satisfiability of each option x_i under the criterion c_j is expressed via intuitionistic fuzzy number (a_{ij}, b_{ij}). Let $w = (w_1, w_2, \cdots, w_n)$ be the weight vector of criteria. Decision maker's goal is to obtain the ranking order of the options $x_1, x_2, \cdots, x_m$.

Next we introduce a ranking method of the options based on IFNs, which involves the following steps:

Step I: By using weighted average operator or weighted geometric mean operator based on IFNs in Xu (2007), we aggregate the elements $\alpha_{ij}\left(j=1,2,\cdots,n\right)$ in row i, and obtain the comprehensive evaluation value α_i of option x_i:

$$\alpha_i = f_w\left(\alpha_{i1},\alpha_{i2},\cdots,\alpha_{in}\right) = \sum_{j=1}^{n} w_j \alpha_{ij}$$

or

$$\alpha_i = g_w\left(\alpha_{i1},\alpha_{i2},\cdots,\alpha_{in}\right) = \prod_{j=1}^{n} \alpha_j^{w_j}, i=1,2,\cdots,m.$$

Step II: Compare $\alpha_1, \alpha_2, \cdots, \alpha_m$ by using the possibility degree method for ranking IFNs.

Example 3.1. Assume that there are 5 criteria for evaluation of candidates for senior positions: morality(c_1), job attitude (c_2), work style(c_3), knowledge structure (c_4) and leadership(c_5). The weight vector of the criteria is:

$$w = \left(0.20, 0.10, 0.25, 0.30, 0.15\right)^T.$$

Suppose there are 5 candidates, x_1, x_2, x_3, x_4 and x_5. The evaluation information under the criteria is represented by IFNs. The corresponding decision making matrix is as follows,

$$D = \begin{pmatrix} (0.3,0.5) & (0.2,0.6) & (0.6,0.1) & (0.2,0.4) & (0.1,0.8) \\ (0.1,0.7) & (0.1,0.8) & (0.6,0.3) & (0.8,0.1) & (0.2,0.7) \\ (0.4,0.3) & (0.7,0.1) & (0.2,0.6) & (0.2,0.7) & (0.2,0.6) \\ (0.1,0.7) & (0.1,0.8) & (0.1,0.7) & (0.2,0.7) & (0.8,0.1) \\ (0.4,0.5) & (0.7,0.2) & (0.3,0.3) & (0.1,0.7) & (0.1,0.8) \end{pmatrix}$$

Now we rank the candidates $x_1, x_2, \cdots, x_5$ by the method addressed in Section 2.2.

Using weighted average operator for IFNs, we obtain comprehensive evaluation values for 5 candidates, $\alpha_1 = \left(0.333, 0.3417\right)$, $\alpha_2 = \left(0.5402, 0.3202\right)$, $\alpha_3 = \left(0.3153, 0.4573\right)$, $\alpha_4 = \left(0.3067, 0.5298\right)$ and $\alpha_5 = \left(0.3017, 0.4766\right)$.

By Step 1 and Step 2 in the possibility degree method for ranking IFNs, we obtain the possibility degree matrix P and the preference relation matrix M:

$$P = \begin{pmatrix} 0.5 & 0.254 & 0.6206 & 0.7193 & 0.6519 \\ 0.746 & 0.5 & 0.9932 & 1 & 1 \\ 0.3794 & 0.0068 & 0.5 & 0.6037 & 0.5366 \\ 0.2807 & 0 & 0.3963 & 0.5 & 0.4374 \\ 0.3481 & 0 & 0.4634 & 0.5626 & 0.5 \end{pmatrix},$$

$$M = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

By Step 3 and Step 4 of the possibility degree method, we get the ranking result of the 5 candidates, $x_2 \overset{0.746}{\succ} x_1 \overset{0.6206}{\succ} x_3 \overset{0.5366}{\succ} x_5 \overset{0.5626}{\succ} x_4$.

If we adopt the score function $S\left(\alpha\right) = a - b$ to rank $\alpha_1, \alpha_2, \cdots, \alpha_5$, we get:

$S\left(\alpha_1\right) = -0.0087$,

$S\left(\alpha_2\right) = 0.2200$, $S\left(\alpha_3\right) = -0.1420$,

$S\left(\alpha_4\right) = -0.2231$, $S\left(\alpha_5\right) = -0.1749$.

Then the ranking of 5 candidates is $x_2 \succ x_1 \succ x_3 \succ x_5 \succ x_4$.

Obviously, the rank of the 5 candidates is same by using our possibility degree method and the score function. While the possibility degree method provides more information to the decision makers, as we may be more certain of x_2 is superior to x_1 than that x_3 is superior to x_5.

4. CONCLUSION

With the easier information acquisition, IFNs are used to represent the degree of satisfiability and non-satisfiability of each alternative with respect to a set of criteria for MCDM problems. And then reasonable methods to compare IFSs are studied to rank those alternatives represented by IFNs. In this paper, by defining two possibility degree formulas to compare two IFNs, we propose the method of ranking n IFNs and its application to multi-criteria decision making. For two IFNs, our method brings the same ranking order of IFNs as that derived by the score function defined by Chen and Tan (1994). Moreover, adoption of possibility degree provides additional information for the comparison of IFNs. As to more than two IFNs, our method can do further comparison of the IFNs that are indifferent when only using the possibility degree formula.

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