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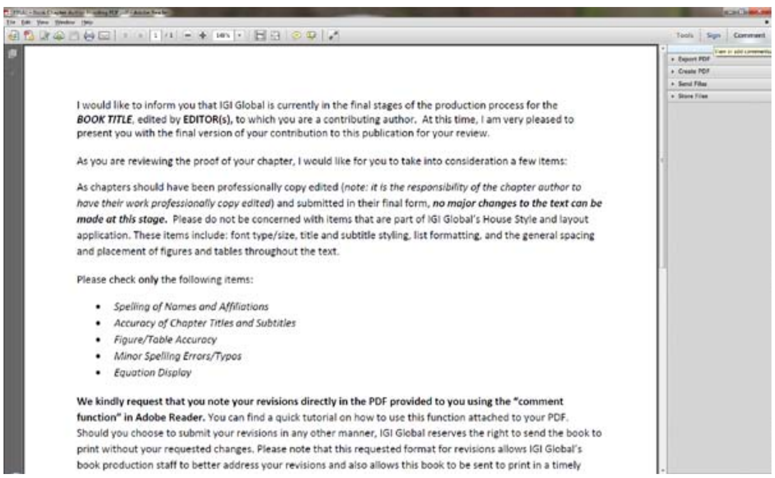

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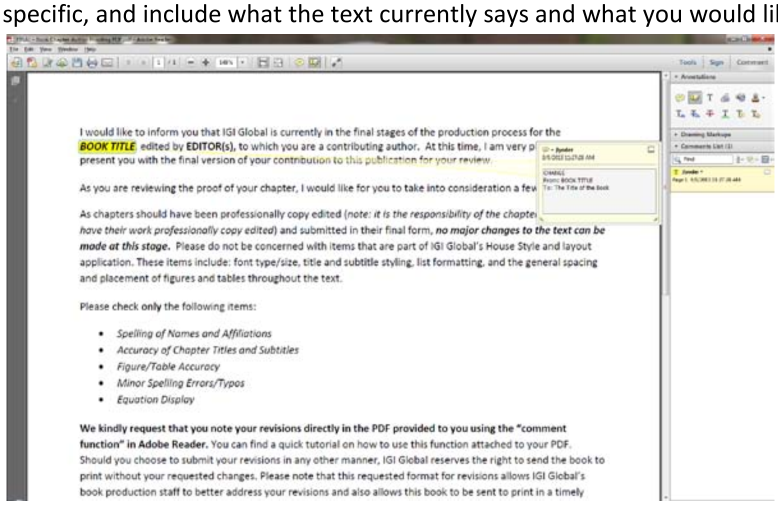

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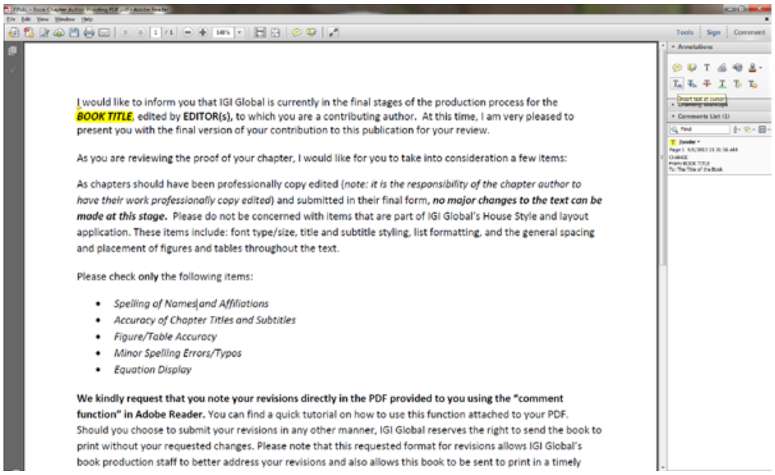

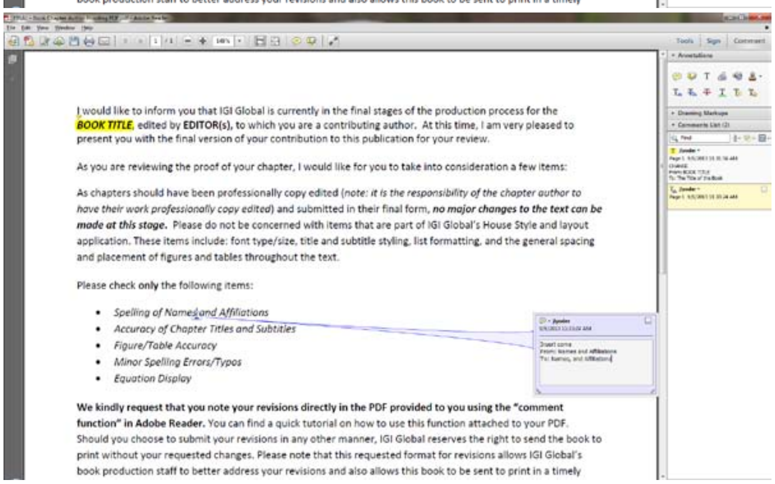

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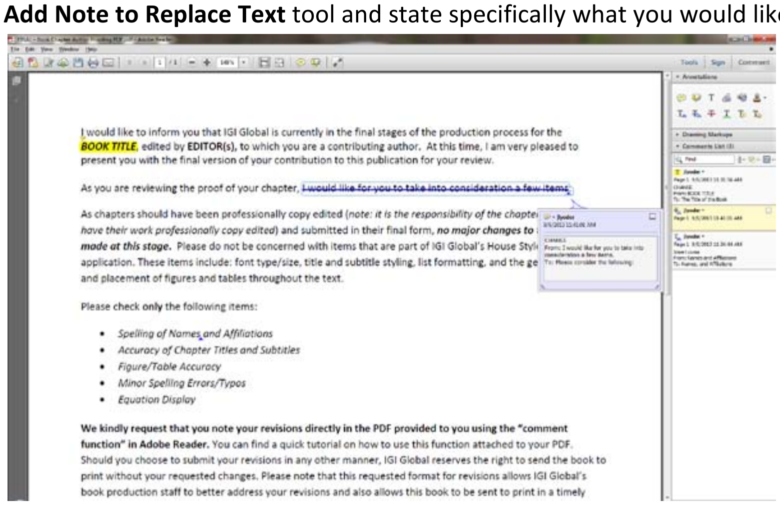

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Linguistic Multi-Attribute Decision Making with a Prioritization Relationship

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ABSTRACT

In this paper we consider linguistic information aggregation problems where a prioritization relationship exists over attributes. The authors define a prioritized 2-tuple ordered weighted averaging (PTOWA) operator to aggregate satisfactions of alternatives under attributes with a linear prioritized ordering. The authors then use the PTOWA and a TOWA operator to aggregate linguistic information where attributes are partitioned into some categories of which prioritization between categories exists. Finally, two illustrative examples are employed to show the feasibility of the proposed method.

Keywords: *2-Tuple, Linguistic Information, Multi-Attribute Decision Making, Prioritized Two-Tuple Ordered Weighted Averaging (PTOWA) Operator, Two-Tuple Ordered Weighted Averaging (TOWA) Operator*

1. INTRODUCTION

Due to the complexity and uncertainty of the objective world, as well as the fuzziness of the human mind, some attributes are more suitable to be evaluated in the form of language (Herrera & Verdegay, 1993; Herrera & Herrera-Viedma, 1997; Torra, 1997; Herrera & Martínez, 2000; Herrera & Martínez, 2001; Xu, 2007; Wei, Feng & Zhang, 2009). For example, when evaluating the comprehensive qualities of the students or the performance of cars, the decision makers prefer to use “excellent”, “good” and “poor” for judgment. For linguistic information aggregation, various linguistic aggregation operators have been proposed, including linguistic OWA operator (Herrera & Verdegay, 1993), induced-linguistic OWA operator (Herrera & Herrera-Viedma, 1997), linguistic WOWA operator (Torra, 1997), etc. In the aggregation process using these operators, the results do not exactly match any of the initial linguistic terms. Therefore, an approximation process is developed to express the results in the initial expression domain, but leads to the loss of information and lack of precision. Herrera and

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Martínez (2000) presented an analytical method based upon 2-tuple for linguistic aggregation. They proposed both 2-tuple weighted average (TWA) operator and 2-tuple ordered weighted averaging (TOWA) operator (Herrera & Martínez, 2000), and then successfully applied the TOWA operator to multigranular hierarchical linguistic contexts in a multi-expert decision making problem (Herrera & Martínez, 2001). Many achievements have been taken in multi-attribute decision making (MADM) with these linguistic aggregation operators.

It is important to see that the above linguistic aggregation operators have the ability to trade off between attributes. While in some situations where a prioritization relationship over the attributes exists, we do not expect this kind of compensation. Yager (2004) studied this kind of problem where decision information is described by real numbers. He pointed out that the importance weights of lower priority attributes were based on the satisfaction of alternative to the higher priority attributes. Based on this idea, Yager proposed the prioritized average (PA) operator (Yager, 2008) and the prioritized ordered weighted averaging (POWA) operator (Yager, 2009). Later Wei and Tang (2012) proposed two averaging operators, a generalized PA operator and a generalized POWA operator. In the case with one attribute in each priority category, both operators reduce to the PA operator and the POWA operator proposed by Yager.

Motivated by the above-mentioned studies, we consider linguistic aggregation problems where a prioritization relationship exists over the attributes. This paper is structured as follows. In Section 2, we make a brief review of 2-tuple and its related operators. In Section 3, we propose a prioritized 2-tuple ordered weighted averaging (PTOWA) operator and discuss its properties. We then use this operator and a TOWA operator to aggregate satisfactions of attributes by alternatives. The paper is concluded in Section 4.

2. 2-TUPLE LINGUISTIC REPRESENTATION MODEL AND TOWA OPERATORS

For MADM problems with some qualitative attributes, we need to use a linguistic term set to describe the decision information. Herrera and Martínez (2000) introduced a finite and totally ordered discrete linguistic term set: $S = \left\{ s_\alpha \middle| \alpha = 0, 1, \cdots, \tau \right\}$, whose cardinality value is odd. For example, a set of seven linguistic terms s could be the equation shown in Box 1.

Furthermore, Herrera defined 2-tuple to aggregate linguistic information:

Definition 1: (Herrera & Martínez, 2001) Let $S = \left\{ s_0, s_1, \cdots, s_\tau \right\}$ be a linguistic term set, then the 2-tuple can be obtained by the translation function θ:

$$\theta : S \to S \times \left[-0.5, 0.5 \right), \theta\left(s_i \right) = \left(s_i, 0 \right), \text{for any } s_i \in S \tag{1}$$

Definition 2: (Herrera & Martínez, 2001) Let $S = \left\{ s_0, s_1, \cdots, s_\tau \right\}$ be a linguistic term set, $s_i = S$ and $\beta \in \left[0, \tau \right]$, a value representing the result of a symbolic aggregation operation, then the 2-tuple can be obtained with the following function:

Box 1.

$$S = \begin{Bmatrix} s_0 = \text{extremely poor},\ s_1 = \text{very poor}, s_2 = \text{poor}, s_3 = \text{fair}, \\ s_4 = \text{good}, s_5 = \text{very good}, s_6 = \text{extremely good} \end{Bmatrix}$$

$$\Delta : \left[0,\tau\right] \rightarrow S \times \left[-0.5, 0.5\right),\ \Delta\left(\beta\right) = \left(s_i, \alpha\right) = \begin{cases} s_i,\ i = round\left(\beta\right) \\ \alpha = \beta - i,\ \ i \in \left[-0.5, 0.5\right) \end{cases} \quad (2)$$

where round $\left(\bullet\right)$ is the usual round operation:

Definition 3: (Herrera & Martínez, 2001) Let $S = \left\{s_0, s_1, \cdots, s_\tau\right\}$ be a linguistic term set, $s_i \in S$ and $\left(s_i, \alpha\right)$ be a 2-tuple. There is always a Δ^{-1} function such that from a 2-tuple it returns its equivalent numerical value $\beta \in \left[0\ \tau\right]$:

$$\Delta^{-1} : S \times \left[-0.5, 0.5\right) \rightarrow \left[0,\tau\right], \Delta^{-1}\left(s_i, \alpha\right) = i + \alpha = \beta \quad (3)$$

Let $\left(s_i, \alpha_1\right)$ and $\left(s_j, \alpha_2\right)$ be two 2-tuples, which have the properties as follows:

1. There exists an order: if $i > j$ then $\left(s_i, \alpha_1\right)$ is bigger than $\left(s_j, \alpha_2\right)$; if $i = j$ then:
   a. If $\alpha_1 = \alpha_2$, then $\left(s_i, \alpha_1\right)$ and $\left(s_j, \alpha_2\right)$ represent the same information;
   b. If $\alpha_1 > \alpha_2$, then $\left(s_i, \alpha_1\right)$ is bigger than $\left(s_j, \alpha_2\right)$;
   c. If $\alpha_1 < \alpha_2$, then $\left(s_i, \alpha_1\right)$ is smaller than $\left(s_j, \alpha_2\right)$;
2. There exists a negative operator: $Neg\left(s_i, \alpha\right) = \Delta\left(\tau - \left(\Delta^{-1}\left(s_i, \alpha\right)\right)\right)$, where $\left(s_i, \alpha\right)$ is an arbitrary 2-tuple, $\tau + 1$ is the cardinality of S, $S = \left\{s_0, s_1, \cdots, s_\tau\right\}$;
3. There exists a minimization and a maximization operator:

$$\max\left\{\left(s_i, \alpha_1\right), \left(s_j, \alpha_2\right)\right\} = \left(s_i, \alpha_1\right), \min\left\{\left(s_i, \alpha_1\right), \left(s_j, \alpha_2\right)\right\} = \left(s_j, \alpha_2\right), if\left(s_i, \alpha_1\right) \geq \left(s_j, \alpha_2\right)$$

Definition 4: (Herrera & Martínez, 2001). Let $\left\{\left(b_1, \alpha_1\right), \left(b_1, \alpha_1\right), \cdots, \left(b_n, \alpha_n\right)\right\}$ be a set of 2-tuples, the 2-tuple ordered weighted averaging (TOWA) operator is defined as:

$$TOWA\left\{\left(b_1, \alpha_1\right), \left(b_2, \alpha_2\right), \cdots, \left(b_n, \alpha_n\right)\right\} = \Delta\left(\sum_{j=1}^{n} w_j \beta_j^*\right) \quad (4)$$

where $w = \left\{w_1, w_2, \cdots, w_n\right\}^T$ is the related weighting vector of TOWA operator, such that $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$. β_j^* is the jth largest of the values β_i and $\beta_i = \Delta^{-1}\left(b_i, \alpha_i\right), i = 1, 2, \cdots, n$.

3. PTOWA OPERATOR FOR LINGUISTIC AGGREGATION WITH ATTRIBUTES OF PRIORITIZATION RELATIONSHIP

In this section, we first define a prioritized 2-tuple ordered weighted averaging (PTOWA) operator to aggregate satisfactions of alternatives under attributes with a linear prioritized ordering. We then use the PTOWA and a TOWA operator to aggregate linguistic information where attributes can be partitioned into some categories of which prioritization between categories exists.

3.1. PTOWA Operator

For a linguistic MADM problem, assume that we have a collection of attributes

$C = \{C_1, C_2, \cdots, C_n\}$ and there is a prioritization between the attributes expressed by the linear ordering $C_1 > C_2 > \cdots > C_n$. For any alternative x and an attribute C_j, we assume that $C_j(x) \in S(x \in X)$ indicates the satisfaction of attribute C_j by alternative x, where $S = \{s_0, s_1, \cdots, s_\tau\}$ is a linguistic term set and τ is an even. For each attribute, we transform $C_j(x)$ into a 2-tuple, denoted by a_j. According to the prioritization relationship between attributes and the satisfactions a_j under those attributes, we first obtain the importance weighting vector $u = (u_1, u_2, \cdots, u_n)^T$ of the attributes. For each attribute we assume T_j is its 2-tuple weight and define it with Equation 5:

$$(i)\, T_1 = (s_\tau, 0); \quad (ii)\, T_j = \min\{T_{j-1}, a_{j-1}\},\ j = 2, 3, \cdots, n \tag{5}$$

Transform T_j into its equivalent value, we get the normalized importance weights:

$$u_j = \frac{\Delta^{-1}(T_j)}{\sum_{j=1}^{n} \Delta^{-1}(T_j)},\ j = 1, 2, \cdots, n \tag{6}$$

Now we obtain the importance weighting vector $u = (u_1, u_2, \cdots, u_n)^T$ of the attributes which reflects the prioritization relationship. For a given alternative x, when using TOWA operator to aggregation its satisfaction to each attribute, we have to consider the importance weight associated with each attribute. Both Yager (1997) and Torra (1997) suggested approaches to conducting the aggregation of this type by using OWA operator. Here we use their idea to apply the TOWA operator to the aggregation of linguistic terms with importance weights associated with each attribute.

Firstly we will derive a weighting vector $v = (v_1, v_2, \cdots, v_n)^T$ including the importance information of the prioritized attributes expressed by the weighting vector $u = (u_1, u_2, \cdots, u_n)^T$. We consider two cases to deriving the weighting vector $v = (v_1, v_2, \cdots, v_n)^T$.

Suppose that the weighting vector of TOWA operator is obtained by a BUM function f, a mapping $f : [0,1] \to [0,1]$ satisfying $f(0) = 0$, $f(1) = 1$ and $f(x) \ge f(y)$ if $x > y$. We assume $ind(j)$ is the index of the jth largest of a_j. Thus $a_{ind(j)}$ is the jth largest of a_j and $u_{ind(j)}$ is its associated importance weight. Next we get the weights v_j that will be used in the aggregation. Let $R_0 = 0$, $R_j = \sum_{k=1}^{j} u_{ind(k)}$, then the weights v_j used in the aggregation is acquired by Equation 7:

$$v_j = f(R_j) - f(R_{j-1}),\ \text{for } j = 1 \text{ to } n \tag{7}$$

In another case, suppose that we start with a weighting vector $w = (w_1, w_2, \cdots, w_n)^T$, such that $w_j \ge 0$ and $\sum_{j=1}^{n} w_j = 1$, corresponding to the TOWA operator. We modify these weights w_j ($j = 1, 2, \cdots, n$) to include the weighting vector $u = (u_1, u_2, \cdots, u_n)^T$ of the prioritized attributes. Both Yager (1997) and Torra (1997) suggested modeling a BUM function as a piecewise linear function. It is suggested that the function f interpolates the points $\left(\frac{i}{n}, \sum_{j<i} w_j\right)$. With this, we acquire:

$$f(x) = \sum_{k=1}^{j-1} w_k + w_j\left(nx - (j-1)\right), \frac{j-1}{n} \le x \le \frac{j}{n} \quad (8)$$

Using this function and Equation 7 we obtain the modified weights $v_j \left(j = 1, 2, \cdots, n\right)$.

The modified weights $v_j \left(j = 1, 2, \cdots, n\right)$ take into account both w_j and individual importance weights u_j of the attributes. Then we use the modified weights to aggregate the satisfactions of an alternative under attributes. We define a function as follows:

Definition 5: Let $a_j = \left(b_j, \alpha_j\right)$ $\left(j = 1, 2, \cdots, n\right)$ be the satisfactions of attributes C_j by an alternative, and there is a prioritization between the attributes expressed by the order $C_1 > C_2 > \cdots > C_n$. The prioritized 2-tuple ordered weighted averaging (PTOWA) operator is defined as:

$$PTOWA\left\{\left(b_1, \alpha_1\right), \left(b_2, \alpha_2\right), \cdots, \left(b_n, \alpha_n\right)\right\} = \Delta\left(\sum_{j=1}^{n} v_j \Delta^{-1}\left(a_{ind(j)}\right)\right) \quad (9)$$

where $a_{ind(j)}$ represents the jth largest of a_j, $v = \left(v_1, v_2, \cdots, v_n\right)^T$ is the related weighting vector of PTOWA operator satisfying $v_j \ge 0 \left(j = 1, 2, \cdots, n\right)$ and $\sum_{j=1}^{n} v_j = 1$, and v can be obtained by Equation 7.

For convenience of notation, we denote:

$$PTOWA\left\{\left(b_1, \alpha_1\right), \left(b_2, \alpha_2\right), \cdots, \left(b_n, \alpha_n\right)\right\} = \left(\tilde{b}, \tilde{a}\right)$$

We can easily prove that the PTOWA operator satisfies the following properties:

Proposition 1: (Boundedness) Let $\left\{\left(b_1, \alpha_1\right), \left(b_2, \alpha_2\right), \cdots, \left(b_n, \alpha_n\right)\right\}$ be a set of 2-tuples, then we have:

$$\min_j \left\{\left(b_j, \alpha_j\right)\right\} \le \left(\tilde{b}, \tilde{\alpha}\right) \le \max_j \left\{\left(b_j, \alpha_j\right)\right\}$$

Proposition 2: (Idempotency) Let $\left\{\left(b_1, \alpha_1\right), \left(b_2, \alpha_2\right), \cdots, \left(b_n, \alpha_n\right)\right\}$ be a set of 2-tuples, if $\left(b_j, \alpha_j\right) = \left(b, \alpha\right), j = 1, 2, \cdots, n$. Then we obtain $\left(\tilde{b}, \tilde{\alpha}\right) = \left(b, \alpha\right)$:

Example 1: We assume there are three attributes C_1, C_2 and C_3 with the priority ordering: $C_1 > C_2 > C_3$ and the linguistic term set S is defined as shown in Box 2.

Assume for alternative x we have $C_1(x) = s_3$, $C_2(x) = s_5$ and $C_3(x) = s_2$.

First we conduct the PTOWA aggregation for the alternative x. We transform $C_j(x), j = 1, 2, 3,$ into the form of 2-tuple denoted by $a_1 = \left(s_3, 0\right)$, $a_2 = \left(s_5, 0\right)$ and $a_3 = \left(s_2, 0\right)$, respectively. Then we get $T_1 = \left(s_6, 0\right)$, $T_2 = \min\left\{T_1, a_1\right\} = \left(s_3, 0\right)$,

Box 2.

$$S = \begin{Bmatrix} s_0 = extremely\ poor, s_1 = very\ poor, s_2 = poor, s_3 = fair, \\ s_4 = good, s_5 = very\ good, s_6 = extremely\ good \end{Bmatrix}$$

$T_3 = \min\{T_2, a_2\} = (s_3, 0)$. By Equation 6, we get $u_1 = 0.5$, $u_2 = u_3 = 0.25$.

From the ordering of the satisfactions $a_1 > a_2 > a_3$, we have:

$ind(1) = 2$, $ind(2) = 1$, $ind(3) = 3$

$a_{ind(1)} = (s_5, 0)$, $a_{ind(2)} = (s_3, 0)$,
$a_{ind(3)} = (s_2, 0)$
$u_{ind(1)} = 0.25$, $u_{ind(2)} = 0.5$, $u_{ind(3)} = 0.25$

Thus:

$R_0 = 0$, $R_1 = 0.25$, $R_2 = 0.75$, $R_3 = 1$

Assume the aggregation is guided by the function $f(x) = x^2$. By Equation 7, we obtain the weights:

$$v_1 = f(R_1) - f(R_0) = 0.0625$$

$$v_2 = f(R_2) - f(R_1) = 0.5$$

and:

$$v_3 = f(R_3) - f(R_2) = 0.4375$$

Then by Equation 9, we get the overall degree of satisfaction of alternative x:

$$(\tilde{s}, \tilde{\alpha}) = \Delta\left(\sum_{j=1}^{n} w_j \times \Delta^{-1}\left(a_{ind(j)}\right)\right)$$
$$= \Delta(2.6875) = (s_3, -0.3125)$$

Assume the scope of the aggregation is expressed by an importance weights vector $w = (0.2, 0.3, 0.5)$ of TOWA, by Equation 8, we get the function f such that:

$$f(x) = \begin{cases} 0.6x, & 0 \le x \le \dfrac{1}{3} \\ 0.2 + 0.3(3x - 1), & \dfrac{1}{3} < x \le \dfrac{2}{3} \\ 0.5 + 0.5(3x - 2), & \dfrac{2}{3} < x \le 1 \end{cases}$$

Using this we get $f(R_0) = 0$, $f(R_1) = 0.15$, $f(R_2) = 0.625$ and $f(R_3) = 1$. Then by Equation 7, we get our modified weights $v_1 = 0.15$, $v_2 = 0.475$ and $v_3 = 0.375$. From this we get he overall degree of satisfaction of alternative x:

$$(\tilde{s}, \tilde{\alpha}) = \Delta\left(\sum_{j=1}^{n} v_j \times \Delta^{-1}\left(a_{ind(j)}\right)\right)$$
$$= \Delta(2.925) = (s_3, -0.075)$$

3.2. An Aggregation Algorithm Using PTOWA and TOWA Operators

Above we consider the situation that there is a prioritization between the attributes expressed by the linear ordering $C_1 > C_2 > \cdots > C_n$. Furthermore we assume that the collection $C = \{C_1, C_2, \cdots, C_n\}$ of attributes is partitioned into q distinct categories, $H_1, H_2, \cdots, H_q$ such that $H_i = \{C_{i1}, C_{i2}, \cdots, C_{in_i}\}$. Here C_{ij} are the attributes in category H_i, $C = \bigcup_{i=1}^{q} H_i$ and $n = \sum_{i=1}^{q} n_i$. We assume a prioritization exists between these categories with $H_1 > H_2 > \cdots > H_n$. The attributes in the class H_i have a higher priority than those in H_k if $i < k$. Assume that for any alternative $x \in X$ and each attribute C_{ij}, we have one linguistic term $C_{ij}\left(x\right) \in S$ indicating its satisfaction to attribute C_{ij}.

Next we present one algorithm to aggregate the satisfactions of attributes by alternative x based on the PTOWA operator and the TOWA operator:

Step 1: Aggregate the satisfactions of each category H_i based on the TOWA operator. For each attribute, we transform $C_{ij}\left(x\right)$ into a 2-tuple, denoted by a_{ij}. We associate each priority class H_i with a TOWA weighting vector $W_i = \left(w_{i1}, w_{i2}, \cdots, w_{in_i}\right)^T$, such that $w_{ij} \geq 0$ and $\sum_{j=1}^{n_i} w_{ij} = 1$. Using this we calculate the aggregation value a_i of each category H_i:

$$a_i = TOWA\left(a_{i1}, a_{i2}, \cdots, a_{in_i}\right) = \Delta\left(\sum_{j=1}^{n_i} w_{ij}\beta_{ij}^*\right)$$

where β_{ij}^* is the jth largest of the values β_{ik} and $\beta_{ik} = \Delta^{-1}\left(a_{ik}\right), \ k = 1, 2, \cdots, n_i$:

Step 2: Calculate the importance weight of each category H_i by Equation 5 and Equation 6;

Step 3: Calculate the PTOWA aggregation value for alternative x:

$$PTOWA\left(a_1, a_2, \cdots, a_q\right) = \Delta\left(\sum_{j=1}^{q} v_j \Delta^{-1}\left(a_{ind(j)}\right)\right)$$

where $v = \left(v_1, v_2, \cdots, v_n\right)^T$ is the related weighting vector of PTOWA operator. Then we can use the PTOWA aggregation value to rank the alternatives:

Example 2: Consider the following prioritized collection of attributes: $H_1 = \left\{C_{11}, C_{12}\right\}$, $H_2 = \left\{C_{21}\right\}$ and $H_3 = \left\{C_{31}, C_{32}, C_{33}\right\}$. A prioritization ordering $H_1 > H_2 > H_3$ exists between these categories; the linguistic term set S is defined as shown in Box 3.

Assume for alternative x we have:

$$C_{11}\left(x\right) = s_3, \ C_{12}\left(x\right) = s_4,$$
$$C_{21}\left(x\right) = s_6, \ C_{31}\left(x\right) = s_3,$$
$$C_{32}\left(x\right) = s_4, \ C_{33}\left(x\right) = s_1$$

We now use the above algorithm to aggregate the satisfactions of attributes for alternative x. We associate each priority class H_i with an OWA weighting vector W_i as follows:

$$W_1 = \left(1\right), W_2 = \left(0.5, 0.5\right), W_3 = \left(\frac{1}{6}, \frac{2}{3}, \frac{1}{6}\right)$$

Box 3.

$$S = \begin{Bmatrix} s_0 = extremely\ poor,\ s_1 = very\ poor,\ s_2 = poor,\ s_3 = fair, \\ s_4 = good,\ s_5 = very\ good,\ s_6 = extremely\ good \end{Bmatrix}$$

For priority class H_i, by Step 1, we can get the TOWA aggregation values:

$$a_1 = (s_3, 0)$$
$$a_2 = TOWA\{(s_4, 0), (s_6, 0)\} = (s_5, 0)$$

and:

$$a_3 = TOWA\{(s_3, 0), (s_4, 0), (s_1, 0)\} = (s_2, 0.83)$$

By Step 2, we get:

$$T_1 = (s_6, 0),$$
$$T_2 = \min\{T_1, a_1\} = (s_3, 0),$$
$$T_3 = \min\{T_2, a_2\} = (s_3, 0)$$

With this we have $u_1 = 0.5, u_2 = u_3 = 0.25$.

Assume the scope of the aggregation is expressed by a weighting vector $w = (0.2, 0.3, 0.5)^T$ of TOWA. By the process similar to Example 1, we get the modified weights $v_1 = 0.15, v_2 = 0.475, v_3 = 0.375$.

By Step 3, we get the overall satisfaction $(\tilde{b}, \tilde{\alpha})$ of alternative x:

$$(\tilde{b}, \tilde{\alpha}) = PTOWA(a_1, a_2, a_3)$$
$$= \Delta\left(\sum_{j=1}^{3} v_j \times \Delta^{-1}\left(a_{ind(j)}\right)\right)$$
$$= \Delta(3.236) = (s_3, 0.236)$$

4. CONCLUDING REMARKS

For linguistic aggregation problems where there exists a prioritization relationship between the attributes, we propose a prioritized 2-tuple ordered weighted averaging (PTOWA) operator. Based on the PTOWA operator and the TOWA operator, we give a method to aggregate the satisfactions of an alternative when attributes are partitioned into some categories and there exists a prioritization between categories.

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