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A Novel Linguistic Group Decision-Making Model Based on Extended Hesitant Fuzzy Linguistic Term Sets

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Hesitant fuzzy linguistic term set (HFLTS) is a set with ordered consecutive linguistic terms, and is very useful in addressing the situations where people are hesitant in providing their linguistic assessments. Wang [H. Wang, Extended hesitant fuzzy linguistic term sets and their aggregation in group decision making, *International Journal of Computational Intelligence Systems* **8**(1) (2015) 14–33.] removed the consecutive condition to introduce the notion of extended HFLTS (EHFLTS). The generalized form has wider applications in linguistic group decision-making. By introducing distance measures for EHFLTSs, in this paper we develop a novel multi-criteria group decision making model to deal with hesitant fuzzy linguistic information. The model collects group linguistic information by using EHFLTSs and avoids the possible loss of information. Moreover, it can assess the importance weights of criteria according to their subjective and objective information and rank alternatives based on the rationale of TOPSIS. In order to illustrate the applicability of the proposed algorithm, two examples are given and comparisons are made with the other existing methods.

Keywords: Multi-criteria group decision making; extended hesitant fuzzy linguistic term sets; distance measures; TOPSIS.

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1. Introduction

In multi-criteria decision making (MCDM), many criteria are of qualitative nature, and it is more suitable to evaluate them in the form of languages. For example, when evaluating the reliability of an information system, experts prefer to use fuzzy languages such as "excellent", "good" or "poor" etc. Hence, it is very natural and important to study the fuzzy linguistic approach.[46] Up to now, many linguistic models have been proposed to extend and improve the fuzzy linguistic approach in information modeling and computing processes. There are two classical linguistic computational models in specialized literatures:[33] the semantic model[2,8,30,44] and the symbolic model.[9,11,13,14] Moreover, these models have been successfully applied to many areas, such as, decision making,[4,10,15,45] information retrieval,[16,20] supply chain management.[6]

In these linguistic models, an expert usually uses a single linguistic term in a linguistic term set to assess a linguistic variable. However, when the expert is thinking of several terms at the same time or looking for a more complex linguistic term not usually defined in the linguistic term set, it is difficult for him/her to provide a single term as an expression of his/her knowledge. In order to model such situations, Rodríguez *et al.*[31] used the idea in defining hesitant fuzzy sets[35,36] (see also Refs. 24 and 25) to introduce the concept of hesitant fuzzy linguistic term sets (HFLTSs). So, by means of the HFLTS model, an expert could assess a linguistic variable by using several linguistic terms for decision-making. About the HFLTS theory, some preliminary results have been obtained. Liao *et al.*[26] gave the distance and similarity measures of HFLTSs and applied them to MCDM problems. Zhu and Xu[47] introduced the concept of hesitant fuzzy linguistic preference relations(HFLPRs), developed some consistency measures and two optimization methods to improve the consistency of HFLPRs. Liu *et al.*[28] discussed the additive consistency of linguistic fuzzy preference relations with elements being comparative linguistic expressions. Rodríguez *et al.*[32] presented a linguistic group decision making model capable of dealing with comparative linguistic expressions as preference assessments in hesitant decision situations. Lee and Chen[22] proposed a fuzzy decision making method based on likelihood-based comparison relations of HFLTSs. Beg and Rashid[1] put forward a fuzzy TOPSIS method to aggregate the opinions of experts represented by HFLTSs. Liao *et al.*[27] introduced the hesitant fuzzy linguistic VIKOR method and implemented it into decision making. Liu and Rodríguez[29] constructed the fuzzy envelope of an HFLTS using a fuzzy membership function and then combined with the fuzzy TOPSIS model to solve supplier selection and MCDM problems. Wei *et al.*[42] constructed possibility degree formulas for comparing HFLTSs and defined two aggregation operators to deal with MCDM problems. Wang *et al.*[37] proposed an outranking approach by integrating both HFLTSs and ELECTRE I to solve MCDM problems.

An HFLTS on a linguistic term set S is a subset with ordered consecutive linguistic terms in S, and has been applied in decision-making problems. In the study of HFLTSs, people find that the theory and applications have certain limitations.

For example, let $S = \{s_0$: nothing, s_1: very low, s_2: low, s_3: medium, s_4: high, s_5: very high, s_6: perfect$\}$ be a linguistic term set. We use the scale S to assess the safety of a car. Suppose there are two experts, one assigns s_3 or s_4 and the other s_6. Then we can use a nonconsecutive linguistic term set $\{s_3, s_4, s_6\}$ to describe the opinions of the two experts. This nonconsecutive linguistic term set is called by Wang[40] an extended hesitant fuzzy linguistic term set (EHFLTS). In fact, it is better to use EHFLTSs to aggregate the group linguistic assessments than to use traditional processing models based on aggregation operators in such case where experts are of equal importance. We now compare the aggregation result derived from linguistic operators with that represented by the EHFLTS $\{s_3, s_4, s_6\}$. If we utilize the HLWA operator defined in Ref. 42 to aggregate the 2-expert's evaluations stand for by $\{s_3, s_4\}$ and $\{s_6\}$, then the collective evaluation is $\{s_5\}$, which is a compromise of the three possible linguistic terms s_3, s_4 and s_6. Using $\{s_5\}$ to represent the comprehensive evaluation information of the two experts is obviously not suitable. The aggregation process of the operator brings the loss of information. Beg and Rashid[1] also proposed an operator for aggregating the opinions of experts. By means of this operator, the comprehensive evaluation information is represented by an linguistic interval $[s_4, s_6]$. Using continuous linguistic intervals to represent the aggregation results of discrete linguistic terms is by nature inappropriate. The EHFLTS $\{s_3, s_4, s_6\}$ involves all the possible evaluations of experts, and thus avoids the loss and distortion of information in the intermediate course of information processing.

EHFLTS is a very effective tool to collect the linguistic information of a group, more theory and methods need to be developed. Wang[40] defined some basic operations and two types of operators to aggregate EHFLTSs. These operators are applied to solving hesitant fuzzy linguistic group decision making problems. In this process, the virtual linguistic terms are necessarily introduced in operation and comparison and the obtained aggregation values are relatively tedious. For example, if the evaluations of an alternative under three criteria with an importance weighting vector (0.2,0.3,0.5) are $\{s_2, s_3, s_4\}$, $\{s_4, s_6\}$ and $\{s_1, s_3\}$, respectively, then by the EHFLWA operators[40] the overall evaluation of the alternative is a set with 10 virtual linguistic terms: $\{s_{2.1}, s_{2.3}, s_{2.5}, s_{2.7}, s_{3.1}, s_{3.3}, s_{3.5}, s_{3.7}, s_{3.9}, s_{4.1}\}$. If there are more criteria, then the obtained overall aggregated value is a set with more virtual linguistic terms, which is shown in Example 1. Moreover, a comparison method must be chosen to compare these overall aggregated values of alternatives. In Ref. 40, the mean values of the virtual linguistic term sets are used to rank different virtual linguistic term sets. In this paper, we try to develop a novel method to solve hesitant fuzzy group decision-making problems. The method can avoid the adoption of virtual linguistic terms, tedious aggregated values and the choice of comparison method for virtual linguistic term sets. We first introduce an axiomatic definition of the distance measure for EHFLTSs and three concrete distance formulas. Then we develop a group decision-making model to deal with hesitant fuzzy linguistic information. The model can collect the group linguistic information by using EHFLTSs and avoid the possible loss of information. Moreover, it can assess the importance weights of criteria according to their subjective and objective information and rank alternatives based on the rationale of TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) in Ref. 17. Since an HFLTS is a special EHFLTS, the method are also suitable for HFLTSs.

The paper is organized as follows. Section 2 reviews the definition and basic operations of EHFLTSs. Section 3 gives an axiomatic definition of the distance measure for EHFLTSs and three concrete distance formulas. In Sec. 4, based on the proposed distance measures for EHFLTSs and the rationale of TOPSIS, a model is developed to solve group decision making problems with hesitant fuzzy linguistic information. The model is specified by three phases illustrated by Subsecs. 4.1, 4.2 and 4.3, respectively. Examples in Sec. 5 are given to illustrate the process of Algorithm I and the results are compared with those obtained by other existing methods. Conclusions are drawn in Sec. 6.

2. Extended HFLTSs and Basic Operations

Consider a finite and totally ordered linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$ with odd cardinality and the mid term representing an assessment of "approximately 0.5", and with the rest of the terms being placed symmetrically around it as in Refs. 2, 8, 9, 12 and 45. For example, a set S with seven terms could be given as follows: $S = \{s_0$: nothing, s_1: very low, s_2: low, s_3: medium, s_4: high, s_5: very high, s_6: perfect$\}$. Moreover, it is usually required that the linguistic term set should satisfy the following additional characteristics.

(1) There is a negation operator: $Neg(s_i) = s_{g-i}$, where $g+1$ is the cardinality of the term set;

(2) The set is ordered: $s_i \leq s_j \Longleftrightarrow i \leq j$. Therefore, there exists a maximization operator: $\max(s_i, s_j) = s_i$ if $s_j \leq s_i$, and a minimization operator: $\min(s_i, s_j) = s_i$ if $s_i \leq s_j$.

Definition 1.[31] Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. An HFLTS H_S on S is a subset with ordered and consecutive linguistic terms in S.

Wang[40] generalized the definition of HFLTSs as follows.

Definition 2.[40] Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. An extended HFLTS (EHFLTS) H_S on S is a subset with ordered linguistic terms in S.

We suppose that the elements in an EHFLTS are arranged in increasing order. Obviously, an HFLTS is a special EHFLTS in which the linguistic terms are ordered and consecutive.

For EHFLTSs, Wang[40] defined the "$\vee$" and "$\wedge$" operations, which are called max-union and min-intersection operations in this paper, respectively.

Definition 3. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. For EHFLTSs H_S, H_S^1 and H_S^2 on S,

(1) $\{s_{g-i} \mid s_i \in H_S\}$ is the negation of H_S, denoted by $Neg(H_S)$;

(2) $\{\max\{s_i, s_j\} \mid s_i \in H_S^1, s_j \in H_S^2\}$ is the max-union of H_S^1 and H_S^2, denoted by $H_S^1 \vee H_S^2$;

(3) $\{\min\{s_i, s_j\} \mid s_i \in H_S^1, s_j \in H_S^2\}$ is the min-intersection of H_S^1 and H_S^2, denoted by $H_S^1 \wedge H_S^2$.

Example 1. Let $S = \{s_0$: nothing, s_1: very low, s_2: low, s_3: medium, s_4: high, s_5: very high, s_6: perfect$\}$ be a linguistic term set; $H_S^1 = \{s_2, s_3, s_5\}$ and $H_S^2 = \{s_4, s_5\}$ be two EHFLTSs on S. Then, by Definition 3, we have

$$Neg(H_S^1) = \{s_{6-5}, s_{6-3}, s_{6-2}\} = \{s_1, s_3, s_4\}\,,$$

$$\begin{aligned} H_S^1 \vee H_S^2 &= \{\max\{s_2, s_4\}, \max\{s_2, s_5\}, \max\{s_3, s_4\}, \max\{s_3, s_5\}, \\ &\quad \max\{s_5, s_4\}, \max\{s_5, s_5\}\} \\ &= \{s_4, s_5\}, \end{aligned}$$

and

$$\begin{aligned} H_S^1 \wedge H_S^2 &= \{\min\{s_2, s_4\}, \min\{s_2, s_5\}, \min\{s_3, s_4\}, \min\{s_3, s_5\}, \\ &\quad \min\{s_5, s_4\}, \min\{s_5, s_5\}\} \\ &= \{s_2, s_3, s_4, s_5\}. \end{aligned}$$

Remark 1. For HFLTSs, the results of the above operations are also HFLTSs. In fact, for two HFLTSs, H_S^1 and H_S^2, assume that $H_S^{2+} \leq H_S^{1+}$, where $H_S{}^+ = \max\{s_i \mid s_i \in H_S\}$ and $H_S{}^- = \min\{s_i \mid s_i \in H_S\}$ for an arbitrary HFLTS H_S. Suppose $I(s_i) = i$ for any linguistic term s_i. Then

$$H_S^1 \vee H_S^2 = \begin{cases} H_S^1, & H_S^{2-} \leq H_S^{1-}\,, \\ \{s_i \mid i \in \{I(H_S^{2-}), I(H_S^{2-}) + 1, \ldots, I(H_S^{1+})\}\}, & H_S^{2-} > H_S^{1-}\,, \end{cases}$$

$$H_S^1 \wedge H_S^2 = \begin{cases} H_S^2, & H_S^{2-} \leq H_S^{1-}\,, \\ \{s_i \mid i \in \{I(H_S^{1-}), I(H_S^{1-}) + 1, \ldots, I(H_S^{2+})\}\}, & H_S^{2-} > H_S^{1-}\,. \end{cases}$$

The following property can be derived from Definition 3.

Property 1.[40] Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, H_S, H_S^1, H_S^2 and H_S^3 be four EHFLTSs on S. Then the followings are true:

(1) $Neg(Neg(H_S)) = H_S$.

(2) $Neg(H_S^1 \vee H_S^2) = Neg(H_S^1) \wedge Neg(H_S^2)$ and $Neg(H_S^1 \wedge H_S^2) = Neg(H_S^1) \vee Neg(H_S^2)$.

(3) *Commutativity*: $H_S^1 \vee H_S^2 = H_S^2 \vee H_S^1$ and $H_S^1 \wedge H_S^2 = H_S^2 \wedge H_S^1$.

(4) *Associativity*: $H_S^1 \vee (H_S^2 \vee H_S^3) = (H_S^1 \vee H_S^2) \vee H_S^3$ and $H_S^1 \wedge (H_S^2 \wedge H_S^3) = (H_S^1 \wedge H_S^2) \wedge H_S^3$.

(5) *Distributivity*: $H_S^1 \wedge (H_S^2 \vee H_S^3) = (H_S^1 \wedge H_S^2) \vee (H_S^1 \wedge H_S^3)$ and $H_S^1 \vee (H_S^2 \wedge H_S^3) = (H_S^1 \vee H_S^2) \wedge (H_S^1 \vee H_S^3)$.

3. Distance Measures for EHFLTSs

Xu and Xia[43] defined the distance measures for hesitant fuzzy sets and hesitant fuzzy elements. Liao *et al.*[26] introduced the axiomatic definition of the distance measure and some distance formulas for HFLTSs. In this section, we generalize the definition and formulas, and give the axiomatic definition of the distance measure for EHFLTSs and three concrete distance measures.

Let $l(H_S)$ be the number of linguistic terms in EHFLTS H_S and $s_{H_S}^k$ be the kth smallest linguistic term in H_S. In general case, for two EHFLTSs H_S^1 and H_S^2, $l(H_S^1) \neq l(H_S^2)$. Let $l = \max\{l(H_S^1), l(H_S^2)\}$. In order to operate correctly, we may extend the shorter one until the lengths of both are the same. The best way to extend the shorter one is to add the same linguistic term several times in it until the changed linguistic term set has the same length as the longer one. We may add any linguistic term in the shorter one to extend it. The added linguistic term can be obtained by the following method.

Suppose that H_S^2 is the shorter one, ${H_S^2}^+ = \max\{s_i \mid s_i \in H_S^2\}$, ${H_S^2}^- = \min\{s_i \mid s_i \in H_S^2\}$ and $\xi (0 \leq \xi \leq 1)$ is an optimized parameter. Then the added linguistic term s in H_S^2 can be obtained by

$$s = C^2(\xi, {H_S^2}^+, 1-\xi, {H_S^2}^-) = \xi \odot {H_S^2}^+ \oplus (1-\xi) \odot {H_S^2}^-,$$

where $C^2(\xi, {H_S^2}^+, 1-\xi, {H_S^2}^-)$ is the convex combination of the two linguistic terms ${H_S^2}^+$ and ${H_S^2}^-$ defined in Ref. 9.

For example, let $S = \{s_0$: nothing, s_1: very low, s_2: low, s_3: medium, s_4: high, s_5: very high, s_6: perfect$\}$ be a linguistic term set, $H_S^1 = \{s_1, s_2, s_4, s_5\}$ and $H_S^2 = \{s_2, s_3, s_4\}$ be two EHFLTSs on S. It is noted that $l(H_S^1) > l(H_S^2)$ from which we should extend H_S^2 by adding a linguistic term several times until it has the same length as H_S^1 so as to calculate the distance between H_S^1 and H_S^2. The selection of this linguistic term mainly relies on the decision makers' risk attitudes, which determine the optimized parameter ξ. The optimists expect desirable outcomes and may select $\xi = 1$ and extend H_S^2 as $H_S^2 = \{s_2, s_3, s_4, s_4\}$, and pessimists expect unfavorable outcomes and extend it as $H_S^2 = \{s_2, s_2, s_3, s_4\}$. If the decision makers are neutral and select $\xi = 0.5$, then the added linguistic term s is s_3 and H_S^2 is extended as $H_S^2 = \{s_2, s_3, s_3, s_4\}$. Although different operations may result in different results, this is reasonable because the decision maker's risk attitudes do have a direct influence on the final decision.

For two EHFLTSs H_S^1 and H_S^2 with the same length l, $H_S^1 = H_S^2$ if and only if $s^k_{H_S^1} = s^k_{H_S^2}$, $k = 1, 2, \ldots, l$, where $s^k_{H_S^i}$ is the kth smallest linguistic term in H_S^i for $i = 1, 2$.

Definition 4. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, H_S^1 and H_S^2 be two EHFLTSs on S. Then the distance measure between H_S^1 and H_S^2, denoted by $d(H_S^1, H_S^2)$, should satisfy the following properties:

(1) $0 \leq d(H_S^1, H_S^2) \leq 1$;

(2) $d(H_S^1, H_S^2) = 0$ if and only if $H_S^1 = H_S^2$;

(3) $d(H_S^1, H_S^2) = d(H_S^2, H_S^1)$.

Based on the well-known Hamming distance, the Euclidean distance, the Hausdorff metric and Definition 4, we define the following distance measures for EHFLTSs H_S^1 and H_S^2 with the same length l:
a normalized Hamming distance measure for EHFLTSs:

$$d_1(H_S^1, H_S^2) = \frac{1}{l} \sum_{k=1}^{l} \frac{|I(s^k_{H_S^1}) - I(s^k_{H_S^2})|}{g}, \tag{1}$$

a normalized Euclidean distance measure for EHFLTSs:

$$d_2(H_S^1, H_S^2) = \left(\frac{1}{l} \sum_{k=1}^{l} \left(\frac{I(s^k_{H_S^1}) - I(s^k_{H_S^2})}{g} \right)^2 \right)^{\frac{1}{2}} \tag{2}$$

and a normalized Hausdorff distance measure for EHFLTS:

$$d_3(H_S^1, H_S^2) = \max_k \frac{|I(s^k_{H_S^1}) - I(s^k_{H_S^2})|}{g}, \tag{3}$$

where $I(s_i) = i$ and g is determined by the linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$.

Obviously, the above three distance measures satisfy the relationship:

$$d_1(H_S^1, H_S^2) \leq d_2(H_S^1, H_S^2) \leq d_3(H_S^1, H_S^2).$$

Remark 2. Since HFLTSs are special EHFLTSs, the formulas (1), (2) and (3) can be used to measure the distance between two HFLTSs H_S^1 and H_S^2. In Ref.,[26] the normalized Hamming distance, Euclidean distance and Hausdorff distance between two HFLTSs H_S^1 and H_S^2 are also defined, respectively:

$$d_1(H_S^1, H_S^2) = \frac{1}{l} \sum_{k=1}^{l} \frac{|I(s^k_{H_S^1}) - I(s^k_{H_S^2})|}{g+1},$$

$$d_2(H_S^1, H_S^2) = \left(\frac{1}{l} \sum_{k=1}^{l} \left(\frac{I(s^k_{H_S^1}) - I(s^k_{H_S^2})}{g+1} \right)^2 \right)^{\frac{1}{2}},$$

$$d_3(H_S^1, H_S^2) = \max_k \frac{|I(s^k_{H_S^1}) - I(s^k_{H_S^2})|}{g+1}.$$

Obviously, the distances defined in Ref.[26] are all less than 1 and the distances defined by formulas (1), (2) and (3) are all no more than 1.

4. A Model Dealing with Hesitant Fuzzy Linguistic Group Decision-Making Problems

A multi-criteria linguistic group decision-making problem considered in this paper can be described as follows: let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of alternatives. Suppose there are t evaluators or experts $d_1, d_2, \ldots, d_t$ to provide evaluations of alternatives x_i $(i = 1, 2, \ldots, n)$ under criteria c_j $(j = 1, 2, \ldots, m)$ by a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$. Suppose that the evaluation information of the kth expert is represented by a hesitant fuzzy linguistic decision matrix $(H_S^{ij^{(k)}})_{n\times m}$, denoted by R_k, where each $H_S^{ij^{(k)}}$ is an HFLTS or a single linguistic term (which can be regarded as a special HFLTS) in S, and represents the linguistic assessment provided by the expert d_k for the alternative x_i with respect to the criterion c_j. Decision makers' goal is to obtain the ranking order of the alternatives.

As it was mentioned in Ref. 11, there are two basic approaches to obtain the overall aggregated values of alternatives. One is a direct approach:

$$\{R_1, R_2, \ldots, R_t\} \to solution.$$

According to this method, a solution is derived on the basis of individual decision matrices. The other is an indirect approach:

$$\{R_1, R_2, \ldots, R_t\} \to R \to solution$$

providing a solution on the basis of an overall decision matrix. In what follows, we are going to consider an indirect method dealing with the above linguistic group decision making problem. The method is specified by the following phases: (1) Aggregate the hesitant fuzzy linguistic information of evaluators. (2) Assign the importance weights of criteria based on distance measures. (3) Rank alternatives based on the rationale of TOPSIS. The following subsections will describe these phases in detail.

4.1. *Aggregate the hesitant fuzzy linguistic information of evaluators*

In the above linguistic group decision making problem, the HFLTSs $H_S^{ij^{(k)}}$ represent the evaluation information of the kth expert for the alternative x_i with respect to the criterion c_j. We will use EHFLTSs to collect the evaluations of t experts. Let $H_S^{ij} = \bigcup_{k=1}^{t} H_S^{ij^{(k)}}$ and, for any two HFLTSs H_S^1 and H_S^2, $H_S^1 \bigcup H_S^2 = \{s_i | s_i \in H_S^1 \text{ or } s_i \in H_S^2\}$. Then H_S^{ij} is an EHFLTS and represents the collective evaluation of t experts for the alternative x_i under the criterion c_j. From the hesitant fuzzy linguistic decision matrices $R_k = (H_S^{ij^{(k)}})_{n\times m}$, $k = 1, 2, \ldots, t$, we can construct a collective decision matrix $R = (H_S^{ij})_{n\times m}$.

In comparison of the above method with the existing indirect methods for constructing the collective decision matrix R, the indirect methods use linguistic operators to aggregate individual decision matrices R_k, while in our construction of R, we use EHFLTSs to collect evaluations of all the experts. Our method can eliminate the aggregation step on individual decision matrices. Moreover, the obtained EHFLTS decision matrix involves all the possible evaluations of experts, and thus avoids the possible loss of information. In addition, experts can express their opinions by flexible forms such as single linguistic terms or HFLTSs, which is beneficial for experts to make evaluations in real applications. We also note that the process doesn't consider the relative importance of experts and is suitable for the situation where experts are of equal importance, such as anonymous evaluations. The detailed comparison combined with examples with the existing methods will be made in Subsec. 5.1.

4.2. *Assign the objective importance weights of criteria based on distance measures*

In Subsec. 4.1, we get the collective decision matrix R. In order to rank the alternatives, the relative importance weights of criteria need to be considered and incorporated into the evaluations of alternatives under criteria. These weights may play a dominant role toward the final ranking of the alternatives. In general, the weights of criteria are predefined according to an expert's or a decision-maker's knowledge, which are regarded as one kind of subjective weights of the criteria. Compared with the predefined weights, the evaluation information of alternatives under each criterion may reflect its relative importance in a more objective sense. The weights of criteria derived from evaluation information are referred as the objective weights of criteria.

We now develop a method to determine the objective weights of criteria from the collective decision matrix $R = (H_S^{ij})_{n\times m}$. For a criterion c_j, we use distances between the evaluation values $H_S^{ij}(i = 1, 2, \ldots, n)$ of alternatives to reflect the deviation degree of these evaluation values under this criterion. Then the bigger the deviation under the criterion, the more important the criterion acts for the overall aggregation values. So we should assign a bigger weight toward this criterion. According to the above analysis, we give the following steps to assess the objective weights of criteria.

Approach to assessing the objective weights of criteria is as follows:

1. Calculate the distances $d(H_S^{ij}, H_S^{kj})$ between H_S^{ij} and H_S^{kj} $(1 \le i, k \le n)$ by Formula (1), (2) or (3).
2. Calculate the objective weights w_j^1 of the criteria c_j $(j = 1, 2, \ldots, m)$:

$$\omega_j^1 = \frac{\sum_{i=1}^{n}\sum_{k=1}^{n} d(H_S^{ij}, H_S^{kj})}{\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1}^{n} d(H_S^{ij}, H_S^{kj})} = \frac{\sum_{i=1}^{n-1}\sum_{k=i+1}^{n} d(H_S^{ij}, H_S^{kj})}{\sum_{j=1}^{m}\sum_{i=1}^{n-1}\sum_{k=i+1}^{n} d(H_S^{ij}, H_S^{kj})}. \quad (4)$$

4.3. *Rank alternatives based on the rationale of TOPSIS*

TOPSIS developed by Hwang and Yoon[17] is a practical technique to solve MCDM problems. It is based on the idea that the alternative having the shortest distance from the positive ideal solution, and, on the other hand, the farthest distance from the negative ideal solution is the optimal alternative. TOPSIS has the following advantages: (1) having intuitive geometric significance; (2) making relatively sufficient use of the original data and having a less information loss. Therefore, TOPSIS has been successfully applied to many areas, such as, supplier selection,[3,6] transportation,[18] human resource management,[7] product design[21] and so on. In addition, the rationale of TOPSIS has been used to deal with different decision-making information, such as linguistic information,[41] interval data,[19] fuzzy numbers,[5,23,38] vague sets[39] and intuitionistic fuzzy sets.[3] In this subsection, we will apply the proposed distance measures of EHFLTSs and the rationale of TOPSIS to ranking alternatives.

Suppose that $w = (w_1, w_2, \ldots, w_m)$ is the weighting vector of criteria, and $R = (H_S^{ij})_{n\times m}$ is the collective decision matrix. Approach to ranking alternatives is as follows:

1. Determine a generalized hesitant fuzzy linguistic positive ideal solution (GHFLPIS) $R^+ = (H_S^{+1}, H_S^{+2}, \ldots, H_S^{+m})$ and a hesitant fuzzy linguistic negative ideal solution (GHFLNIS) $R^- = (H_S^{-1}, H_S^{-2}, \ldots, H_S^{-m})$. The elements H_S^{+j} and H_S^{-j} $(j = 1, 2, \ldots, m)$ are defined as follows:
   $H_S^{+j} = H_S^{1j} \vee H_S^{2j} \vee \cdots \vee H_S^{nj}$ if C_j is a benefit criterion and $H_S^{+j} = H_S^{1j} \wedge H_S^{2j} \wedge \cdots \wedge H_S^{nj}$ if C_j is a cost criterion;
   $H_S^{-j} = H_S^{1j} \wedge H_S^{2j} \wedge \cdots \wedge H_S^{nj}$ if C_j is a benefit criterion and $H_S^{-j} = H_S^{1j} \vee H_S^{2j} \vee \cdots \vee H_S^{nj}$ if C_j is a cost criterion.
2. For alternative x_i, let $R_i = (H_S^{i1}, H_S^{i2}, \ldots, H_S^{im})$, $i = 1, 2, \ldots, n$. Calculate the weighted distances $d(R_i, R^+)$ and $d(R_i, R^-)$:
$$d(R_i, R^+) = \sum_{k=1}^{m} \omega_k d(H_S^{ik}, H_S^{+k}), \quad d(R_i, R^-) = \sum_{k=1}^{m} \omega_k d(H_S^{ik}, H_S^{-k}).$$
3. Calculate the closeness coefficients D_i of alternatives $x_i (i = 1, 2, \ldots, n)$:
$$D_i = \frac{d(R_i, R^+)}{d(R_i, R^-) + d(R_i, R^+)}. \tag{5}$$
4. Rank the alternatives according to the principle that the smaller D_i is, the better the alternative x_i is.

4.4. *An algorithm to deal with hesitant fuzzy linguistic decision information*

From the above discussion about the group decision model, we develop the following Algorithm I to solve the above group decision-making problem with hesitant fuzzy linguistic information.

Algorithm I

Step 1. Based on the hesitant fuzzy linguistic decision matrices $R_k = (H_S^{ij(k)})_{n\times m}$, $k = 1, 2, \ldots, t$, construct a collective decision matrix $R = (H_S^{ij})_{n\times m}$, where $H_S^{ij} = \bigcup_{k=1}^{t} H_S^{ij(k)}$.

Step 2. Use the steps in Subsec. 4.2 to assess the objective weighting vector $w^1 = (w_1^1, w_2^1, \ldots, w_m^1)^T$ of criteria. Suppose the subjective weighting vector $w^2 = (w_1^2, w_2^2, \ldots, w_m^2)^T$ of criteria is given. Integrate the objective weights w_j^1 and the subjective weights w_j^2 into the weights w_j of the criteria c_j:

$$w_j = \gamma w_j^1 + (1-\gamma) w_j^2, \quad \gamma \in [0,1], \quad j = 1, 2, \ldots, m. \tag{6}$$

Suppose the subjective weighting vector $w^2 = (w_1^2, w_2^2, \ldots, w_m^2)^T$ of criteria is completely unknown. Then we can let $\gamma = 1$ in Eq. (6). So the weighting vector $w = (w_1, w_2, \ldots, w_m)^T$ is equal to the objective weighting vector w^1.

Step 3. For the generalized hesitant fuzzy linguistic decision matrix R and the importance weighting vector w of criteria, we can apply the steps in Subsec. 4.3 to ranking the alternatives.

We note that if there is only one evaluator or expert to provide evaluations of alternatives x_i $(i = 1, 2, \ldots, n)$ under criteria c_j $(j = 1, 2, \ldots, m)$, then only Step 2 and Step 3 need to be considered.

5. Illustrative Examples

In this section, the proposed Algorithm I is demonstrated by two illustrative examples and the results are compared with those obtained by the methods in Refs. 1, 29, 31 and 40.

5.1. *A group decision-making example*

Example 1. Let us consider a practical linguistic group decision-making problem. A manufacturing company searches the best global supplier for one of its most critical parts used in assembling process among three suppliers $x_i (i = 1, 2, 3)$. The criteria considered in selection are capacity of the production (c_1), capacity of accuracy (c_2), supplier's credibility (c_3) and cost performance of the product (c_4). Suppose there are three experts d_1, d_2, d_3 to provide evaluations by using the linguistic term set $S = \{s_0$: nothing, s_1: very low, s_2: low, s_3: medium, s_4: high, s_5: very high, s_6: perfect$\}$. The decision matrices $R_k = (H_S^{ij(k)})_{3\times 4}$ $(k = 1, 2, 3)$ are as follows:

$$R_1 = \begin{pmatrix} \{s_4, s_5\} & \{s_3\} & \{s_4\} & \{s_3\} \\ \{s_5\} & \{s_5\} & \{s_3\} & \{s_4\} \\ \{s_4\} & \{s_3\} & \{s_5\} & \{s_3\} \end{pmatrix}, \quad R_2 = \begin{pmatrix} \{s_5, s_6\} & \{s_2\} & \{s_3, s_4\} & \{s_2\} \\ \{s_5\} & \{s_5\} & \{s_3\} & \{s_4\} \\ \{s_3\} & \{s_2, s_3\} & \{s_5\} & \{s_1\} \end{pmatrix},$$

$$R_3 = \begin{pmatrix} \{s_4\} & \{s_2\} & \{s_4\} & \{s_2\} \\ \{s_5\} & \{s_5\} & \{s_3\} & \{s_3\} \\ \{s_3\} & \{s_4\} & \{s_5\} & \{s_1\} \end{pmatrix}.$$

Suppose the subjective weights of the criteria $c_j (j = 1, 2, 3)$ are completely unknown, then the following steps are given to get the ranking order of alternatives.

5.1.1. *Illustration of the proposed algorithm*

By Step 1 in Algorithm I, the overall evaluations $H_S^{ij} (i = 1, 2, 3; j = 1, 2, 3, 4)$ of x_i with respect to c_j are formed directly by the union of three experts' evaluations. For example, three experts' evaluations of alternative x_1 with respect to criterion c_1 are $\{s_4, s_5\}$, $\{s_5, s_6\}$ and $\{s_4\}$, respectively. Then the overall evaluation is formed by $\{s_4, s_5\} \bigcup \{s_5, s_6\} \bigcup \{s_4\}$, which is equal to the EHFLTS $\{s_4, s_5, s_6\}$. So the resultant decision matrix R is as follows:

$$R = \begin{pmatrix} \{s_4, s_5, s_6\} & \{s_2, s_3\} & \{s_3, s_4\} & \{s_2, s_3\} \\ \{s_5\} & \{s_5\} & \{s_3\} & \{s_3, s_4\} \\ \{s_3, s_4\} & \{s_2, s_3, s_4\} & \{s_5\} & \{s_1, s_3\} \end{pmatrix}.$$

By Step 2, we calculate the objective weights of criteria. Suppose we adopt the distance measure d_2 defined by Formula (2) and the decision maker is optimistic. Then we can obtain the distance matrix shown in Table 1:

Table 1. Distance matrix.

	$d_2(H_S^{i1}, H_S^{k1})$	$d_2(H_S^{i2}, H_S^{k2})$	$d_2(H_S^{i3}, H_S^{k3})$	$d_2(H_S^{i4}, H_S^{k4})$
$(i = 1, k = 2)$	0.1361	0.4249	0.1178	0.1667
$(i = 1, k = 3)$	0.2357	0.0962	0.2635	0.1178
$(i = 2, k = 3)$	0.2635	0.3600	0.3333	0.2635

From Formula (4), we obtain the objective weighting vector ω^1 of criteria:

$$\omega^1 = (0.2286, 0.3170, 0.2572, 0.1972)^T.$$

Suppose $\gamma = 1$ in Eq. (6), then the weighting vector ω of criteria is equal to their objective weighting vector ω^1.

By Step 3, we have $R^+ = (\{s_5, s_6\}, \{s_5\}, \{s_5\}, \{s_3, s_4\})$ and $R^- = (\{s_3, s_4\}, \{s_2, s_3\}, \{s_3\}, \{s_1, s_2, s_3\})$. Thus,

$$d_2(R_1, R^+) = 0.2665,\ d_2(R_1, R^-) = 0.1110,\ d_2(R_2, R^+) = 0.1126\,,$$
$$d_2(R_2, R^-) = 0.2519,\ d_2(R_3, R^+) = 0.2423,\ d_2(R_3, R^-) = 0.1352\,.$$

So the closeness coefficients D_i of the three suppliers are: $D_1 = 0.7060, D_2 = 0.3091, D_3 = 0.6419$, and the ranking is $x_2 \succ x_3 \succ x_1$.

5.1.2. *Comparison analysis and discussion*

Three methods in Refs. 1, 40 and 42 are respectively proposed to solve the above group decision-making problem. The method in Ref. 42 is a direct method, while the method in Refs. 1 and 40 are indirect approaches like Algorithm I. Now, we conduct a comparison with the methods in Refs. 1 and 40 based on Example 1.

In Ref. 1, a method based on TOPSIS was proposed to aggregate the evaluation information of experts represented by HFLTSs. By this method, we can get the collective decision matrix R, the positive ideal solution R^+ and the negative ideal solution R^-:

$$R = \begin{pmatrix} [s_4, s_5] & [s_2, s_3] & [s_4, s_4] & [s_2, s_3] \\ [s_5, s_5] & [s_5, s_5] & [s_3, s_3] & [s_3, s_4] \\ [s_3, s_4] & [s_3, s_4] & [s_5, s_5] & [s_1, s_3] \end{pmatrix},$$

$$R^+ = ([s_5, s_6], [s_5, s_5], [s_5, s_5], [s_4, s_4]),$$

$$R^- = ([s_3, s_3], [s_2, s_2], [s_3, s_3], [s_1, s_1]) .$$

For two linguistic intervals I_S^1 and I_S^2, the distance $d(I_S^1, I_S^2)$ is defined by $|q' - q|+|p' - p|$, where $I_S^1 = [s_p, s_q]$ and $I_S^2 = [s_{p'}, s_{q'}]$. By this distance formula, the relative closenesses RC_i, defined by $\frac{d(R_i,R^-)}{d(R_i,R^-)+d(R_i,R^+)}$, of alternatives x_i to the ideal solutions are as follows: $RC_1 = 0.4286$, $RC_2 = 0.7143$, $RC_3 = 0.4762$. So the ranking of three alternatives is $x_2 \succ x_3 \succ x_1$, which is the same as that of Algorithm I.

Beg and Rashid's method in Ref. 1 and the proposed method in this paper are all based on TOPSIS to solve hesitant fuzzy linguistic decision-making problems. However, they differ in the following aspects: the aggregation method of experts' information, the constructions of the positive ideal solution R^+ and the negative ideal solution R^-, and the distance formulas used in the two methods. Beg and Rashid used aggregation operators to derive the evaluation matrix R, the positive ideal solution R^+ and the negative ideal solution R^-. Then the distance formula defined for linguistic intervals is used to rank alternatives. Notably, the elements of the evaluation matrix R, the positive ideal solution R^+ and the negative ideal solution R^- are all linguistic intervals. The transformation from discrete linguistic terms to intervals is by nature inappropriate.

On the other hand, Beg and Rashid's aggregation operator, by which the collective matrix is derived, is insensitive to the change of aggregated elements. In Ref. 1, the collective evaluation matrix $R = ([s_{p_{ij}}, s_{q_{ij}}])$ is derived by the formulas: $s_{p_{ij}} = \min\{\min_{k=1}^t H_S^{ij(k)+}, \max_{k=1}^t H_S^{ij(k)-}\}$ and $s_{q_{ij}} = \max\{\min_{k=1}^t H_S^{ij(k)+}, \max_{k=1}^t H_S^{ij(k)-}\}$. For experts' evaluations $\{s_4, s_5\}$, $\{s_5, s_6\}$ and $\{s_4\}$ to alternative x_1 under criterion c_1 in Example 1, the aggregation result is $[s_4, s_5]$. If we change the evaluations of experts e_1 and e_3 from $\{s_4, s_5\}$ and $\{s_4\}$ to $\{s_2, s_3, s_4, s_5\}$ and $\{s_2, s_3, s_4\}$ or $\{s_1, s_2, s_3, s_4, s_5\}$ and $\{s_0, s_1, s_2, s_3, s_4\}$, respectively, then the aggregation result is still the linguistic interval $[s_4, s_5]$. The great change of two experts' evaluations don't cause the change of the aggregation result. Obviously it is not reasonable.

The proposed method in this paper uses EHFLTSs to get the evaluation matrix that involves all the evaluations of experts and avoids the loss and distortion of information. The weights of criteria can be derived by the subjective and objective information. The positive ideal solution R^+ and the negative ideal solution R^- are obtained by the operations of max-union and min-intersection on HFLTSs. The distance measures for EHFLTSs and TOPSIS are used to rank alternatives.

Wang[40] proposed a method to solve the above linguistic group decision-making problem. Following Wang's method, the weights of criteria need to be given firstly, then the problem can be processed by the following steps.

Step 1. Based on the hesitant fuzzy linguistic decision matrices $R_k = (H_S^{ij(k)})_{n\times m}$, $k = 1, 2, \ldots, t$, construct a collective decision matrix $R = (H_S^{ij})_{n\times m}$, where $H_S^{ij} = \bigcup_{k=1}^{t} H_S^{ij(k)}$.

Step 2. Utilize the EHFLWA operator to obtain the collective evaluation values of the alternatives.

Step 3. Use the expected linguistic terms and hesitation degrees of the collective evaluation values to rank the alternatives.

By these steps and the weighting vector $\omega^1 = (0.2286, 0.3170, 0.2572, 0.1972)^T$ obtained by Algorithm I, we obtain the collective evaluation values of the alternatives:

$$H_1 = \{s_{2.71}, s_{2.91}, s_{2.94}, s_{2.97}, s_{3.03}, s_{3.14}, s_{3.16}, s_{3.17}, s_{3.20}, s_{3.23}, s_{3.26}, s_{3.29}, s_{3.37}, s_{3.40}, s_{3.43}, s_{3.46}, s_{3.48}, s_{3.49}, s_{3.52}, s_{3.63}, s_{3.69}, s_{3.71}, s_{3.75}, s_{3.94}\},$$

$$H_2 = \{s_{4.09}, s_{4.29}\},$$

$$H_3 = \{s_{2.80}, s_{3.03}, s_{3.12}, s_{3.20}, s_{3.35}, s_{3.43}, s_{3.44}, s_{3.51}, s_{3.66}, s_{3.74}, s_{3.83}, s_{4.06}\}.$$

The expected linguistic terms are $E(H_1) = s_{3.33}, E(H_2) = s_{4.19}, E(H_3) = s_{3.43}$, respectively. So the ranking of alternatives is $x_2 \succ x_3 \succ x_1$, which is the same as that obtained by Algorithm I.

Comparing Algorithm I and Wang's method,[40] we find that both use ELFLTSs to collect the linguistic information of a group and obtain a collective decision matrix. But the following steps are completely different from Wang's method. Wang used a given weighting vector of criteria and the EHFLWA operator to aggregate EHFLTSs, and adopted a method based on expected linguistic term and hesitation degree of a virtual linguistic term set to rank alternatives. While Algorithm I uses the distance measures for EHFLTSs and the rationale of TOPSIS to rank alternatives. Moreover, Algorithm I can be used to derive weights of criteria according to the subjective and objective information. In Wang's method, the weights of criteria can be only given. His paper did not give any method to derive weights. We also note from the above computation that, by Wang's operators, the obtained overall evaluation values of alternatives are relatively tedious and a comparison method need be chosen to rank the aggregation values of alternatives. Our proposed algorithm can avoid the tedious aggregation values and the choice of comparison method for the aggregation values, and is more simple and effective.

5.2. *A hesitant fuzzy linguistic decision making example*

We now apply Algorithm I to the example used by Rodríguez *et al.*[31]

Example 2. Let $C = \{c_1, c_2, c_3\}$ be a set of benefit criteria, $X = \{x_1, x_2, x_3\}$ be a set of alternatives and $S = \{s_0$: nothing(n), s_1: very low(vl), s_2: low(l), s_3: medium(m), s_4: high(h), s_5: very high(vh), s_6: perfect(p)$\}$ be the linguistic term set used to generate the linguistic expressions. The assessments given by an expert to the alternatives are shown in Table 2.

Table 2. Assessments provided for the decision problem.

	c_1	c_2	c_3
x_1	between vl and m	between h and vh	h
x_2	between l and m	m	lower than l
x_3	greater than h	between vl and l	greater than h

By the transformation function E_{G_H} defined in,[31] we transform the linguistic expressions into HFLTSs which are shown in Table 3.

Table 3. Assessments transformed into HFLTSs.

	c_1	c_2	c_3
x_1	$\{s_1, s_2, s_3\}$	$\{s_4, s_5\}$	$\{s_4\}$
x_2	$\{s_2, s_3\}$	$\{s_3\}$	$\{s_0, s_1, s_2\}$
x_3	$\{s_4, s_5, s_6\}$	$\{s_1, s_2\}$	$\{s_4, s_5, s_6\}$

Suppose $\omega^2 = (0.2, 0.3, 0.5)^T$ is the subjective weighting vector of the criteria. We now apply Algorithm I to ranking the alternatives.

In Eq. (6) of Algorithm I, we suppose $\gamma = 0$. So the weighting vector ω of criteria is equal to the subjective weighting vector ω^2.

Suppose the decision maker is optimistic. From $R^+ = (\{s_4, s_5, s_6\}, \{s_4, s_5\}, \{s_4, s_5, s_6\})$ and $R^- = (\{s_1, s_2, s_3\}, \{s_1, s_2\}, \{s_0, s_1, s_2\})$, we get the following weighted distances shown in Table 4 by using distance measures d_1, d_2 and d_3, respectively.

Table 4. Weighted distances.

i	$d_1(R_i, R^+)$	$d_1(R_i, R^-)$	$d_2(R_i, R^+)$	$d_2(R_i, R^-)$	$d_3(R_i, R^+)$	$d_3(R_i, R^-)$
1	0.1833	0.4000	0.2076	0.4091	0.2667	0.4833
2	0.4861	0.0972	0.4917	0.1063	0.5333	0.1333
3	0.1500	0.4333	0.1500	0.4333	0.1500	0.4333

According to Formula (5), we calculate the closeness coefficients of the alternative x_i ($i = 1, 2, 3$) and rank the alternatives. The results are exhibited in Table 5.

Table 5. Closeness coefficients and ranking order of alternatives.

i	D_i^1	D_i^2	D_i^3
1	0.3143	0.3366	0.3556
2	0.8333	0.8223	0.8000
3	0.2571	0.2571	0.2571
Rankings	$x_3 \succ x_1 \succ x_2$	$x_3 \succ x_1 \succ x_2$	$x_3 \succ x_1 \succ x_2$

From Table 4, it is noted that for the alternative x_i ($i = 1, 2, 3$), $d_1(R_i, R^+) \leq d_2(R_i, R^+) \leq d_3(R_i, R^+)$ and $d_1(R_i, R^-) \leq d_2(R_i, R^-) \leq d_3(R_i, R^-)$. By using three kinds of distance measures of HFLTSs to respectively calculate the closeness coefficients of the alternative x_i, the obtained results may be different shown in Table 5. However, in three cases, the ranking order of alternatives is the same, which is $x_3 \succ x_1 \succ x_2$.

For Example 2, the method in Ref. 31 didn't consider the weights of criteria or supposed that the criteria have equal importance. By using Algorithm I, we can flexibly handle the weights of criteria, which is very important in solving a decision-making problem.

Liu and Rodríguez[29] also proposed a method based on TOPSIS to solve hesitant fuzzy multi-criteria decision-making problems. We now conduct a comparison with this method.

By the method in Ref. 29, the assessments of experts represented by HFLTSs are translated into fuzzy envelopes represented by trapezoidal fuzzy membership functions, which are shown in Table 6.

Table 6. Assessments transformed into fuzzy envelopes.

	c_1	c_2	c_3
x_1	$(0, 0.298, 0.364, 0.67)$	$(0.5, 0.67, 0.83, 1)$	$(0.33, 0.5, 0.5, 0.67)$
x_2	$(0.17, 0.33, 0.5, 0.67)$	$(0.17, 0.33, 0.33, 0.5)$	$(0, 0, 0.1478, 0.5)$
x_3	$(0.5, 0.8522, 1, 1)$	$(0, 0.17, 0.33, 0.5)$	$(0.5, 0.8522, 1, 1)$

Let $R^+ = ((1,1,1,1), (1,1,1,1), (1,1,1,1), (1,1,1,1))$ and $R^- = ((0,0,0,0), (0,0,0,0), (0,0,0,0), (0,0,0,0))$. By the distance formula $d(A,B) = \frac{1}{4}(|a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2|)$ for two trapezoidal fuzzy numbers $A = (a_1, b_1, c_1, d_1)$ and $B = (a_2, b_2, c_2, d_2)$, the closeness coefficients CC_i, defined by $\frac{d(R_i,R^-)}{d(R_i,R^-)+d(R_i,R^+)}$ of the three alternatives can be obtained as $CC_1 = 0.5416$, $CC_2 = 0.2642, CC_3 = 0.6616$ and the ranking is $x_3 \succ x_1 \succ x_2$, which is the same as that of Algorithm I.

Both Algorithm I and Liu and Rodríguez's method[29] are based on TOPSIS and can be used to solve multi-criteria decision-making problems with the hesitant fuzzy linguistic information, while the basis for the two methods is not alike. Using Liu and Rodríguez's method, we first need to translate an HFLTS into a fuzzy envelope, which is a trapezoidal fuzzy membership function obtained by aggregating the fuzzy membership functions of the linguistic terms in the HFLTS. Then the fuzzy TOPSIS is used to rank alternatives. In the process of the translation from an HFLTS into a fuzzy envelope, the different importance of the linguistic terms in an HFLTS is supposed, and is reflected in the calculation process of the parameters of the trapezoidal fuzzy membership function using the OWA operator. By contrast, Algorithm I deals with linguistic terms in an HFLTS as possible evaluation values with equal importance.

Compared with the considered methods, Algorithm I avoids choosing different operators to aggregate the evaluations represented by HFLTSs or single linguistic terms and does not need to deal with the HFLTSs as linguistic intervals or fuzzy envelopes, so it can avoid the loss and distortion of information. It can also assess the importance weights of criteria according to their subjective and objective information. Consequently, Algorithm I is more flexible and more precise in dealing with linguistic decision making problems.

6. Conclusions

The theory of HFLTSs has wide application prospect in objectively dealing with the situations where people have hesitancy in providing their linguistic assessments, but it has some limitations in describing group linguistic decision making information. Wang[40] introduced the notion of EHFLTSs by removing the consecution condition in HFLTSs, and proposed some operators for EHFLTSs to solve hesitant fuzzy linguistic group decision-making problems. In this paper, we have introduced the axiomatic definition of the distance measure for EHFLTSs and three concrete distance formulas, and then developed a new method to deal with group decision-making problems with hesitant fuzzy linguistic information. By the proposed method, linguistic evaluations of the evaluators are collected by EHFLTSs which could eliminate the aggregation step on individual decision matrices and avoid the possible loss of information; the importance weights of criteria can be assessed according to their subjective and objective information and the alternatives can be ranked based on the rationale of TOPSIS. Even for the multi-criteria decision making problems with hesitant fuzzy linguistic information, the proposed method is also suitable. The comprehensive and detailed comparison analysis with the existing methods has been made and the results have shown that the proposed method is more flexible and effective in managing linguistic decision making problems.

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