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Information aggregation operators based on hesitant fuzzy sets and prioritization relationship

Cuiping Wei[a,*], Zhiliang Ren[b], Xijin Tang[c] and Jiangang Feng[d]
[a]*College of Mathematical Science, Yangzhou University, Jiangsu Yangzhou, China*
[b]*School of Economics and Management, Southeast University, Jiangsu Nanjing, China*
[c]*Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, China*
[d]*Management School, Qufu Normal University, Shandong Rizhao, China*

Abstract. Hesitant fuzzy set (HFS) is a very useful technology in dealing with the situation that decision makers are hesitant among several values when asked to give evaluation information for alternatives. The aim of this paper is to aggregate the hesitant fuzzy information with prioritization relationship. Using the operations on HFSs, we first give a method to model the linear prioritization relationship among the aggregated hesitant fuzzy elements (HFEs) and determine the weighting vector of these elements by the operations of HFEs. Then some novel hesitant fuzzy prioritized aggregation operators are defined based on the ordered weighted average operator, the generalized weighted average operator, the quasi weighted average operator and the ordered modular average operator. Based on the proposed operators, we develop a hesitant fuzzy multi-attribute decision-making (MADM) method. Finally, a real decision problem is provided to illustrate the rationality and effectiveness of proposed operators. Compared with the existing hesitant fuzzy prioritized aggregation operator, the proposed process deriving the weights of the aggregated elements can avoid information loss as far as possible, and the proposed aggregation ways can consider both the aggregation requirements of decision makers and capture the prioritization phenomenon among the aggregated hesitant fuzzy elements.

Keywords: Hesitant fuzzy sets, multi-attribute decision-making, prioritized aggregation operators

1. Introduction

Torra and Narukawa [1] and Torra [2] proposed the concept of hesitant fuzzy sets (HFSs), which allows the membership to have a set of possible values, and some basic operations of HFSs. Then, they studied its relationship with intuitionistic fuzzy sets and fuzzy multisets. Afterwards, in order to aggregate the hesitant fuzzy information, Xia and Xu [3] proposed a series of aggregation operators under various situations and discussed the relationship among them. Then, they applied the developed aggregation operators to solve group decision-making problems with anonymity. Under the assumption that the hesitant fuzzy elements are of the same length for comparison, Xu et al. [4, 5] and Chen et al. [6] defined a variety of distance measures, similarity measures and correlation measures of HFSs. Xu et al. [7] developed some aggregation operators for hesitant fuzzy elements with the aid of quasi-arithmetic means and Choquet integral. Then, they proposed the corresponding decision-making method. Xu and Xia [8]

*Corresponding author. Cuiping Wei, College of Mathematical Science, Yangzhou University, 225002 Jiangsu Yangzhou, China. Tel.:+86 18352787116; E-mail: wei_cuiping@aliyun.com.

introduced the concepts of entropy and cross-entropy for hesitant fuzzy information, and developed several measures formulas of entropy and cross-entropy.

From those results, it is known that hesitant fuzzy set is a very useful tool to deal with uncertainty and some multi-attribute decision-making (MADM) methods have been developed under the hesitant fuzzy environment. However, the above proposed MADM methods are under the assumption that the attributes are at the same priority level. They are characterized by the ability to trade off between attributes. While in some situations where there exists a prioritization relationship over the attributes, we do not want to allow this kind of compensation. Yager first studied this kind of problem with decision information described by real numbers. He pointed out that the importance weights of lower priority attributes were based on the satisfaction of an alternative to the higher priority attribute [9]. Based on this idea, Yager proposed the prioritized average operator, the prioritized "and" operator and the prioritized "or" operator [10]. Yager [11] further proposed the prioritized OWA operator. Chen et al. [12] and Wang et al. [13] found the drawbacks of the method presented in Ref. [9] by some numerical examples and proposed the generalized prioritized MADM method. Although previous researches have greatly developed the priority weighted MADM, there were still some limitations and drawbacks. Yan et al. [14] proposed aggregation operators to overcome the limitations of previous works, and showed the effectiveness and advantages of the proposed approach by comparative analysis with Refs. [12, 13]. Wei and Tang [15] proposed generalized prioritized aggregation operators based on the WOWA operator.

Motivated by the above-mentioned studies, we focus on the information aggregation problem where the satisfactions of attributes by an alternative are described by HFEs and there exists a prioritization relationship over attributes. Wei [16] has studied the problem. He adopted Yager's idea [9] to derive the weights of attributes, and proposed the hesitant fuzzy prioritized weighted average operator and the hesitant fuzzy weighted geometric average operator. It is noted that, in the process of deriving the weights of the attributes (or the arguments), the satisfactions described by HFEs are directly translated to the score values of HFEs. Obviously, the processing method ignores the characteristics of HFEs. So, in this paper, we try to derive the weights of the attributes by the operations of HFEs and avoid information loss as far as possible. Moreover, we find that the proposed hesitant fuzzy prioritized operators are only based on the weighted average operator and the weighted geometric average operator, and can consider only the prioritization relationship. There are many other operators, such as the ordered weighted average (OWA) operator [17, 18], the generalized ordered weighted average (GOWA) operator [19], the quasi weighted average (QWA) operator [20] and the ordered modular average (OMA) operator [21], which are very useful in decision-making [24, 25] and computing systems [26, 29]. These operators are very efficient to reflect the aggregation requirements of decision makers, and can give good inspiration for us to develop the hesitant fuzzy prioritized operators considering both the prioritization relationship and the aggregation requirements of decision makers. From the above analysis, we can set out to our investigation of the hesitant fuzzy prioritized aggregation from two aspects: (1) the improving method to derive the weights of attributes according to their prioritization relationship and (2) the hesitant fuzzy prioritized operators based on the OWA operator, the GWA operator, the QWA operator and the OMA operator.

The reminder of this paper is organized as follows. In Section 2, we review some basic knowledge. We also make a detailed discussion about the properties of the HFWA and HFOWA operators, which is necessary to study the properties of the proposed operators in the next sections. In Section 3, we first propose a method for determining weighting vector of the attributes. Then we define some novel hesitant fuzzy prioritized aggregation operators based on the OWA operator, the GWA operator, the QWA operator and the OMA operator. In Section 4, we develop a method for MADM based on proposed operators for hesitant fuzzy environment. Section 5 gives a practical example and makes a comparative analysis with the existing method. The conclusion is given in Section 6.

For convenience, we list the following notations used in this paper:

i: subscribe index;
j: subscribe index;
X: the discourse set;
x_i: the element in X;
A: hesitant fuzzy set;
$h_A(x_i)$: the set of the possible membership degrees of the element x_i in X to the set A;
h: a hesitant fuzzy element;
H: a collection of hesitant fuzzy elements $h_1, h_2, \cdots, h_n$;
γ_i: any one element in h_i;

$|h|$: the number of values in h;
$s(h)$: the score function of h;
$var(h)$: the variance function of h;
ω_i: the weight of the hesitant fuzzy element h_i;
G: a collection of attributes.

2. Preliminaries

This section reviews some related concepts, such as hesitant fuzzy sets (HESs) [1, 2], hesitant fuzzy weighted average operator and the hesitant fuzzy ordered weighted average operator [3]. The method to compare two hesitant fuzzy elements are also proposed.

Torra [1] extended the fuzzy set to the hesitant fuzzy set (HFS), shown as follows.

Definition 2.1. [1] Let X be a discourse set. A hesitant fuzzy set (HES) on X is defined in terms of a function h that when applied to X it returns a finite subset of [0, 1].

To be easily understood, Xia and Xu [3] expressed an HFS A by

$$A = \{\langle x_i, h_A(x_i)\rangle | x_i \in X\} \tag{2.1}$$

where $h_A(x_i)$ is a set of some values in [0, 1], denoting the possible membership degrees of the element x_i in X to the set A. Especially, if there is only one value in each $h_A(x_i)$ $(i = 1, 2, \ldots, n)$, then the hesitant fuzzy set A reduces to a fuzzy set, which indicates that fuzzy set is a special type of hesitant fuzzy set. Let HFS(X) denote the set of all the HFSs on X.

For convenience, Xia and Xu [3] named $h_A(x_i)$, abbreviated to h, a hesitant fuzzy element (HFE). For any HFE $h = \{\gamma_1, \gamma_2, \cdots, \gamma_{|h|}\}$, we assume that $\gamma_i < \gamma_j$, for $\forall i < j$ $(i, j = 1, 2, \ldots, |h|)$ in the whole paper, where $|h|$ is the number of values in h.

Example 2.1. Let $X = \{x_1,\ x_2,\ x_3\}$ be the discourse set. Then $A = \{\langle x_1, (0.2, 0.3)\rangle, \langle x_2, (0.1, 0.2)\rangle, \langle x_3, (0.6, 0.7, 0.8)\rangle\}$ is an HFS on X.

Torra [2], Xia and Xu [3] gave some operations on HFEs, shown as:

For any three HFEs h, h_1, h_2 and a real number λ,

(1) $h^c = \bigcup_{\gamma\in h} \{1 - \gamma\}$,

(2) $h_1 \cup h_2 = \bigcup_{\gamma_1\in h_1,\gamma_2\in h_2} \{max(\gamma_1, \gamma_2)\}$,

(3) $h_1 \cap h_2 = \bigcup_{\gamma_1\in h_1,\gamma_2\in h_2} \{min(\gamma_1, \gamma_2)\}$,

(4) $h^\lambda = \bigcup_{\gamma\in h} \{\gamma^\lambda\}$,

(5) $\lambda h = \bigcup_{\gamma\in h} \{1 - (1 - \gamma)^\lambda\}$,

(6) $h_1 \oplus h_2 = \bigcup_{\gamma_1\in h_1,\gamma_2\in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$,

(7) $h_1 \otimes h_2 = \bigcup_{\gamma_1\in h_1,\gamma_2\in h_2} \{\gamma_1\gamma_2\}$.

In order to compare HFEs, a method is proposed in [3].

Definition 2.2. [3] For a HFE h, let $|h|$ be the number of values in h. Then $s(h) = \frac{1}{|h|}\sum_{\gamma\in h} \gamma$ is called the score function of h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then h_1 is superior to h_2, denoted by $h_1 \succ h_2$; if $s(h_1) = s(h_2)$, then h_1 is indifferent to h_2, denoted by $h_1 \sim h_2$.

As we can see, the comparison method of HFEs in Definition 2.2 can not distinguish two HFEs with the score value. For example, let $h_1 = \{0.1, 0.2, 0.3, 0.4\}$ and $h_2 = \{0.2, 0.3\}$ be two HFEs. Then $s(h_1) = s(h_2) = 0.25$, that is, h_1 is indifferent to h_2 according to Definition 2.2. However, it is noted that h_1 has more hesitant degree than h_2, and h_2 is superior to h_1 can be more rational. For overcoming the drawback, a new comparison method of HFEs is proposed in [22].

Definition 2.3. [22] Let HFEs $h = \{\gamma_1, \gamma_2, \cdots, \gamma_{|h|}\}$ and $h' = \{\gamma'_1, \gamma'_2, \cdots, \gamma'_{|h'|}\}$, $|h|$ be the number of values in h. Let $var(h) = \frac{\sum_{\gamma\in h}(\gamma-\overline{\gamma})^2}{|h|}$ be the variance function and $s(h) = \frac{1}{|h|}\sum_{\gamma\in h} \gamma$ be the score function of h. Then

(1) If $s(h) > s(h')$, then h is superior to h', denoted by $s(h) \succ s(h')$;
(2) If $s(h) = s(h')$,
  a) if $var(h) < var(h')$, then h is superior to h', denoted by $s(h) \succ s(h')$;
  b) if $var(h) = var(h')$, then h is indifferent to h', denoted by $h \sim h'$.

For the above HFEs $h_1 = \{0.1, 0.2, 0.3, 0.4\}$ and $h_2 = \{0.2, 0.3\}$, $s(h_1) = s(h_2) = 0.25$ and $var(h_1) > var(h_2)$. From Definition 2.3, h_2 is superior to h_1, which is consist with the above analysis.

The new comparison method in Definition 2.3 considers both the mean value and the hesitant degree of a HFE, and is more effective to distinguish HFEs. Therefore, the comparison method will be used in the extended OWA operators proposed in Section 3 for ranking HFEs.

For the aggregation of HFEs, Torra and Narukawa [1] proposed an extended principle on HFEs.

Definition 2.4. [1] Let Θ be a function $\Theta : [0, 1]^n \to [0, 1]$ and let $H = \{h_1, h_2, \cdots, h_n\}$ be a set of n hesitant fuzzy sets on the reference set X. Then, the extension of Θ on H is defined for each x in X by:

$$\Theta_H(x) = \bigcup_{\gamma \in \{h_1(x) \times h_2(x) \times \cdots \times h_n(x)\}} \{\Theta(\gamma)\} \quad (2.2)$$

Based on the above extended principle and the defined operations for HFEs, Xu and Xia [3] gave the hesitant fuzzy weighted average (HFWA) operator and the hesitant fuzzy ordered weighted average (HFOWA) operator [3]:

Let H be a collection of HFEs $h_1, h_2, \cdots, h_n$, the HFWA operator is defined as

$$\begin{aligned} \text{HFWA}(h_1, h_2, \cdots, h_n) &= \bigoplus_{j=1}^{n} (\omega_j h_j) \\ &= \bigcup_{\gamma_1 \in h_1, \cdots, \gamma_n \in h_n} \{1 - \prod_{j=1}^{n} (1 - \gamma_j)^{\omega_j}\} \end{aligned} \quad (2.3)$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is the weighting vector of h_j $(j = 1, 2, \cdots, n)$ with $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$. The HFOWA operator is defined as

$$\begin{aligned} \text{HFOWA}(h_1, h_2, \cdots, h_n) &= \bigoplus_{j=1}^{n} (\omega_j h_{\sigma(j)}) \\ &= \bigcup_{\gamma_1 \in h_{\sigma(1)}, \cdots, \gamma_n \in h_{\sigma(n)}} \{1 - \prod_{j=1}^{n} (1 - \gamma_j)^{\omega_j}\} \end{aligned} \quad (2.4)$$

where $h_{\sigma(j)}$ is the jth largest of the HFEs h_i $(i = 1, 2, \cdots, n)$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is the related weighting vector of the HFOWA operator, such that $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Remark 2.1. It is noted that the HFWA operator and the HFOWA operator don't satisfy the idempotence and the monotonicity. Two counter-examples are given as follows.

Example 2.2. Let $h_1 = h_2 = \{0.1, 0.2\}$ be two HFEs and $\omega = (0.5, 0.5)$ be the weighting vector. Then according to Equation (2.3), we get: $HFWA(h_1, h_2) = 0.5h_1 \oplus 0.5h_2 = \{0.1, 0.1515, 0.1515, 0.2\} \neq h_1$. Thus, the HFWA operator can not satisfy the idempotence.

Example 2.3. Let $h_1 = \{0.3, 0.6\}$, $h_2 = \{0.2\}$, $h_1' = \{0.4, 0.51\}$ and $h_2' = \{0.2\}$. Let $\omega = (0.2, 0.8)$ be the weighting vector. From Definition 2.3, $h_1 \prec h_1'$ and $h_2 \sim h_2'$. According to Equation (2.3), we get:

$HFWA(h_1, h_2) = 0.2h_1 \oplus 0.8h_2 = \{0.2211, 0.3035\}$, $HFWA(h_1', h_2') = 0.2h_1' \oplus 0.8h_2' = \{0.2447, 0.2748\}$. Since $s(HFWA(h_1, h_2)) = 0.2623 > s(HFWA(h_1', h_2')) = 0.2597$, $HFWA(h_1', h_2') \prec HFWA(h_1, h_2)$. Thus, the HFWA operator can not satisfy the monotonicity.

Proposition 2.1. *(Boundedness) Let H be a collection of HFEs $h_1, h_2, \cdots, h_n$. Let h^+ and h^- be the HFE in H with the maximal and minimal score value, respectively. Then*

$$h^- \preceq HFWA(h_1, h_2, \cdots, h_n) \preceq h^+$$

$$h^- \preceq HFOWA(h_1, h_2, \cdots, h_n) \preceq h^+$$

3. Hesitant fuzzy prioritized aggregation operators

The attributes have different priority level in many real decision-making problems. For solving those problems with hesitant fuzzy information, some prioritized aggregation operators are proposed in this section based on the HFWA and HFOWA operators [3], the generalized weighted average operator and the quasi-arithmetic means operator [20].

3.1. The HFPWA and HFPOWA operators

In this subsection, we proposed the HFPWA and HFPOWA operators based on the HFWA and HFOWA operators [3].

Under the hesitant fuzzy environment, suppose that G is a collection of attributes $G_1, G_2, \cdots, G_n$ and there is a prioritized relation between the attributes expressed by the linear ordering $G_1 \succ G_2 \succ \cdots \succ G_n$, which indicates that attribute G_j has a higher priority to G_k if $j < k$. For any alternative x and attribute G_j, we assume that the satisfaction of attribute G_j by alternative x is represented by an HFE $h_j(x)$, abbreviated by h_j.

In order to model the prioritized relationship between attributes, the idea that the weight associated with an attribute depends upon the satisfaction of the higher priority attributes [9, 12, 14] is adopted to obtain the importance weighting vector $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ of the attributes.

Let

$$\begin{aligned} &T_1 = \{1\}, \\ &T_j = T_{j-1} \cap h_{j-1}, \quad j = 2, 3, \cdots, n \\ &l_j = \textstyle\prod_{k=1}^{j} s(T_k), \quad j = 1, 2, \cdots, n \end{aligned} \quad (3.5)$$

Then the importance weights of attributes, that are the weights of the hesitant fuzzy evaluation values h_j $(j = 1, 2, \cdots, n)$, can be calculated by l_i:

$$\omega_j = \frac{l_j}{\sum_{i=1}^{n} l_i}, \quad j = 1, 2, \cdots, n \tag{3.6}$$

Remark 3.1. From Equations (3.5) and (3.6), we can easily get that $\omega_i \geq \omega_j$ if $i < j$. Moreover, l_i associated with the weight of G_i increases with any elements in h_{i-1}. So the weight of an attribute depends upon the satisfaction of the higher priority attribute.

Next, an example will be shown for obtaining the importance weighting vector of the attributes in the situation that the satisfactions are represented by HFEs and compared with the method in Ref. [16].

Example 3.1. Let the satisfactions of attributes $G_j (j = 1, 2, 3, 4)$ by alternative x is as follows:

	G_1	G_2	G_3	G_4
x	{0.2, 0.3}	{0.1, 0.2}	{0.3, 0.4}	{0.4, 0.5}

then, according to Equations (3.5) and (3.6), we can get:

T_1	T_2	T_3	T_4
1	{0.2, 0.3}	{0.1, 0.2}	{0.1, 0.2}
l_1	l_2	l_3	l_4
1	0.25	0.0375	0.005625
ω_1	ω_2	ω_3	ω_4
0.7733	0.1933	0.029	0.0044

The method in [16] to derive the weights shown as follows:

Let

$$T_1^* = 1, \quad T_j^* = \prod_{k=1}^{j-1} s(h_k)\ (j = 2, \cdots, n).$$

Then the importance weights of attributes can be calculated by T_i^*:

$$\omega_j = \frac{T_j^*}{\sum_{i=1}^{n} T_i^*}, \quad j = 1, 2, \cdots, n.$$

According to the method in Ref. [16], we can get:

T_1^*	T_2^*	T_3^*	T_4^*
1	0.25	0.0375	0.013125
ω_1^*	ω_2^*	ω_3^*	ω_4^*
0.7689	0.1922	0.0288	0.0101

The method in [16] derives the weights by translating the HFEs to their score values. However, Equations (3.5) and (3.6) derive the weights by the operation on HFEs. Thus, this process of deriving the weights can avoid loss information as far as possible.

Based on this weight-determined technics proposed in this section, we define the following hesitant fuzzy prioritized aggregation operators.

Definition 3.1. Let H be a collection of HFEs $h_1, h_2, \cdots, h_n$. Then

(1) the hesitant fuzzy prioritized weighted average (HFPWA) operator is defined as

$$\begin{aligned} &\text{HFPWA}(h_1, h_2, \cdots, h_n) \\ &= \omega_1 h_1 \oplus \omega_2 h_2 \oplus \cdots \oplus \omega_n h_n = \bigoplus_{j=1}^{n} \left(\omega_j h_j\right) \end{aligned} \tag{3.7}$$

where the weights ω_i $(i = 1, 2, \cdots, n)$ of HFEs is derived by Equation (3.6).

(2) the hesitant fuzzy prioritized ordered weighted average (HFPOWA) operator is defined as

$$\begin{aligned} &\text{HFPOWA}(h_1, h_2, \cdots, h_n) \\ &= \frac{\omega_{\sigma(1)} v_1}{\sum_{i=1}^{n} \omega_{\sigma}(i) v_i} h_{\sigma(1)} \oplus \cdots \oplus \frac{\omega_{\sigma(n)} v_n}{\sum_{i=1}^{n} \omega_{\sigma(i)} v_i} h_{\sigma(n)} \\ &= \bigoplus_{j=1}^{n} \left(\frac{\omega_{\sigma(j)} v_j}{\sum_{i=1}^{n} \omega_{\sigma(i)} v_i} h_{\sigma(j)} \right) \end{aligned} \tag{3.8}$$

where ω_i $(i = 1, 2, \cdots, n)$ can be obtained by Equation (3.6) and $v = (v_1, v_2, \cdots, v_n)$ is the related weighting vector of the HFPOWA operator, and $h_{\sigma(j)}$ is the jth largest of h_i $(i = 1, 2, \cdots, n)$, $\omega_{\sigma(j)}$ is the weight of $h_{\sigma(j)}$.

Proposition 3.1. *Let H be a collection of HFEs $h_1, h_2, \cdots, h_n$. Then*

(1) the aggregated value by using the HFPWA operator is also a HFE, and

$$\begin{aligned} &HFPWA(h_1, h_2, \cdots, h_n) \\ &= \frac{l_1}{\sum_{i=1}^{n} l_i} h_1 \oplus \frac{l_2}{\sum_{i=1}^{n} l_i} h_2 \oplus \cdots \oplus \frac{l_n}{\sum_{i=1}^{n} l_i} h_n \\ &= \bigoplus_{j=1}^{n} \left(\frac{l_j}{\sum_{i=1}^{n} l_i} h_j \right) \\ &= \bigcup_{\gamma_1 \in h_1, \cdots, \gamma_n \in h_n} \left\{ 1 - \prod_{j=1}^{n} (1 - \gamma_j)^{\frac{l_j}{\sum_{i=1}^{n} l_i}} \right\} \end{aligned} \tag{3.9}$$

where l_j is calculated by Equation (3.5).

(2) the aggregated value by using the HFPOWA operator is also a HFE, and

$$HFPOWA(h_1, h_2, \cdots, h_n) = \bigcup_{\gamma_1 \in h_{\sigma(1)}, \cdots, \gamma_n \in h_{\sigma(n)}} \left\{ 1 - \prod_{j=1}^{n} (1-\gamma_j)^{\frac{\omega_{\sigma(j)} v_j}{\sum_{i=1}^{n} \omega_{\sigma(i)} v_i}} \right\} \tag{3.10}$$

where ω_i $(i = 1, 2, \cdots, n)$ can be obtained by Equation (3.6) and $v = (v_1, v_2, \cdots, v_n)$ is the related weighting vector of the HFPOWA operator, and $h_{\sigma(j)}$ is the jth largest of h_i $(i = 1, 2, \cdots, n)$, $\omega_{\sigma(j)}$ is the weight of $h_{\sigma(j)}$.

Proposition 3.2. *Let H be a collection of HFEs $h_1, h_2, \cdots, h_n$. Let h^+ and h^- be the HFE in H with the maximal and minimal score value, respectively. Then*

$$h^- \preceq HFPOWA(h_1, h_2, \cdots, h_n) \preceq h^+$$

and

$$h^- \preceq HFPOWA(h_1, h_2, \cdots, h_n) \preceq h^+$$

From Remark 2.1, it is obvious that the HFPWA and HFPOWA operators don't satisfy the idempotence and the monotonicity. That is, if $h_j = h$, for all j, then $\text{HFPWA}_\lambda(h_1, h_2, \cdots, h_n) = h$ and $\text{HFPOWA}_\lambda(h_1, h_2, \cdots, h_n) = h$ are not necessarily true. In addition, if $H = \{h_1, h_2, \cdots, h_n\}$ is a collection of HFEs, $H' = \{h'_1, h'_2, \cdots, h'_n\}$ is also a collection of HFEs, and $h_j \preceq h'_j$ $(j = 1, 2, \cdots, n)$, then the fact that $\text{HFPWA}_\lambda(H) \preceq \text{HFPWA}_\lambda(H')$ and $\text{HFPOWA}_\lambda(H) \preceq \text{HFPOWA}_\lambda(H')$ are not necessarily true.

Many approaches have been developed for determining the associated weighting vector $v = (v_1, v_2, \cdots, v_n)$ of the OWA operator, which were made a detailed overview in [23]. Different methods reflect different attitudes of a decision-maker or his/her requirements for aggregated arguments. These approaches are effective for determining the weighting vector associated to the HFPOWA operator. So the advantage of using the HFPOWA operator is that it considers both the prioritization relationship of attributes and the decision-maker's requirements for aggregated HFEs.

3.2. Extended hesitant fuzzy prioritized aggregation operators

Based on the generalized weighted average operator, the quasi-arithmetic means operator [20] and the ordered modular averages (OMAs) [21], we propose some extended hesitant fuzzy prioritized aggregation operators (including the GHFPWA operator, the GHFPOWA operator, the QHFPWA operator, the QHFPOWA operator and the HFPMWA operator), and study their properties.

Definition 3.2. Let H be a collection of HFEs $h_1, h_2, \cdots, h_n$, $\lambda > 0$, and l_j $(j = 1, 2, \cdots, n)$ be defined by Equation (3.5). Then a generalized hesitant fuzzy prioritized weighted average (GHFPWA) operator is defined as

$$\text{GHFPWA}_\lambda(h_1, h_2, \cdots, h_n) = \left(\frac{l_1}{\sum_{i=1}^{n} l_i}(h_1)^\lambda \oplus \frac{l_2}{\sum_{i=1}^{n} l_i}(h_2)^\lambda \oplus \cdots \oplus \frac{l_n}{\sum_{i=1}^{n} l_i}(h_n)^\lambda \right)^{\frac{1}{\lambda}} = \bigcup_{\gamma_1 \in h_1, \cdots, \gamma_n \in h_n} \left\{ \left(1 - \prod_{j=1}^{n} (1-\gamma_j^\lambda)^{\frac{l_j}{\sum_{i=1}^{n} l_i}} \right)^{\frac{1}{\lambda}} \right\} \tag{3.11}$$

where l_j $(j = 1, 2, \cdots, n)$ is defined by Equation (3.5).

Definition 3.3. Let H be a collection of HFEs $h_1, h_2, \cdots, h_n$, $\lambda > 0$. Then a generalized hesitant fuzzy prioritized weighted average (GHFPOWA) operator is defined as

$$\text{GHFPOWA}_\lambda(h_1, h_2, \cdots, h_n) = \left(\bigoplus_{j=1}^{n} \left(\frac{\omega_{\sigma(j)} v_j}{\sum_{i=1}^{n} \omega_{\sigma(i)} v_i} (h_{\sigma(j)})^\lambda \right) \right)^{\frac{1}{\lambda}} = \bigcup_{\gamma_1 \in h_{\sigma(1)}, \cdots, \gamma_n \in h_{\sigma(n)}} \left\{ \left(1 - \prod_{j=1}^{n} (1-\gamma_{\sigma(j)}^\lambda)^{\frac{\omega_{\sigma(j)} v_j}{\sum_{i=1}^{n} \omega_{\sigma(i)} v_i}} \right)^{\frac{1}{\lambda}} \right\} \tag{3.12}$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is defined by Equation (3.6), $v = (v_1, v_2, \cdots, v_n)$ is the related weighting vector of the GHFPOWA operator, and $h_{\sigma(j)}$ is the *jth* largest of h_i $(i = 1, 2, \cdots, n)$, $w_{\sigma(j)}$ is the weight of $h_{\sigma(j)}$.

Proposition 3.3. *Let H be a collection of HFEs $h_1, h_2, \cdots, h_n$. Then*

(1) If $\lambda = 1$, then the GHFPWA operator can be reduced to the HFPWA operator;

if $\lambda = 1$, then the GHFPOWA operator can be reduced to the HFPOWA operator.

(2) Boundedness: Let H be a collection of HFEs $h_1, h_2, \cdots, h_n$, h^+ and h^- be the HFE in H with the maximal and minimal score value, respectively. Then

$$h^- \preceq GHFPWA(h_1, h_2, \cdots, h_n) \preceq h^+$$

and

$$h^- \preceq GHFPOWA(h_1, h_2, \cdots, h_n) \preceq h^+$$

Remark. The GHFPWA operator and the GHFPOWA operator can not meet the idempotency and the monotonicity.

Based on the hesitant fuzzy prioritized operators and the quasi-arithmetic means [20], we can get the quasi hesitant fuzzy prioritized operators.

Definition 3.4. Let H be a collection of HFEs $h_1, h_2, \cdots, h_n$, l_j $(j = 2, \cdots, n)$ be defined by Equation (3.5). Then a quasi hesitant fuzzy prioritized weighted average (QHFPWA) operator is defined as

$$\begin{aligned} &\mathrm{QHFPWA}(h_1, h_2, \cdots, h_n) \\ &= f^{-1}\left(\bigoplus_{j=1}^{n}\left(\frac{l_j}{\sum_{i=1}^{n} l_i} f(h_j)\right)\right) \\ &= \bigcup_{\gamma_1 \in h_1, \cdots, \gamma_n \in h_n}\left\{f^{-1}\left(1 - \prod_{j=1}^{n}(1 - f(\gamma_j))^{\frac{l_j}{\sum_{i=1}^{n} l_i}}\right)\right\} \end{aligned} \tag{3.13}$$

where $f : [0, 1] \to [0, 1]$ is a strictly continuous monotonic function.

Definition 3.5. Let H be a collection of HFEs $h_1, h_2, \cdots, h_n$. Then a quasi hesitant fuzzy prioritized weighted average (QHFPOWA) operator is defined as

$$\begin{aligned} &\mathrm{QHFPOWA}(h_1, h_2, \cdots, h_n) \\ &= f^{-1}\left(\bigoplus_{j=1}^{n}\left(\frac{\omega_{\sigma(j)} v_j}{\sum_{i=1}^{n} \omega_{\sigma(i)} v_i} f(h_{\sigma(j)})\right)\right) \\ &= \bigcup_{\gamma_1 \in h_{\sigma(1)}, \cdots, \gamma_n \in h_{\sigma(n)}}\left\{f^{-1}\left(1 - \prod_{j=1}^{n}(1 - f(\gamma_j))^{\frac{\omega_{\sigma(j)} v_j}{\sum_{i=1}^{n} \omega_{\sigma(i)} v_i}}\right)\right\} \end{aligned} \tag{3.14}$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is defined by Equation (3.6), $v = (v_1, v_2, \cdots, v_n)$ is the related weighting vector of the QHFPOWA operator, $h_{\sigma(j)}$ is the jth largest of h_j $(j = 1, 2, \cdots n)$, $\omega_{\sigma(j)}$ is the weight of $h_{\sigma(j)}$ and $f : [0, 1] \to [0, 1]$ is a strictly continuous monotonic function.

Proposition 3.4. *If $f(x) = x$, then the QHFPWA operator can be reduced to the HFPWA operator, and the QHFPOWA operator can be reduced to the HFPOWA operator. If $f(x) = x^\lambda$, $\lambda > 0$, then the QHFPWA operator becomes the GHFPWA operator, and the QHFPOWA operator becomes the GHFPOWA operator.*

Based on the ordered modular averages (OMAs) [21], we can further generalize the hesitant fuzzy prioritized operator as follows:

Definition 3.6. Let H be a collection of HFEs $h_1, h_2, \cdots, h_n$, and l_j $(j = 1, 2, \cdots, n)$ be defined by Equation (3.5). Then the hesitant fuzzy prioritized modular weighted average operator is defined as

$$\begin{aligned} &\mathrm{HFPMWA}(h_1, h_2, \cdots, h_n) \\ &= \frac{l_1}{\sum_{i=1}^{n} l_i} f_1(h_1) \oplus \frac{l_2}{\sum_{i=1}^{n} l_i} f_2(h_2) \oplus \cdots \oplus \frac{l_n}{\sum_{i=1}^{n} l_i} f_n(h_n) \\ &= \bigoplus_{j=1}^{n}\left(\frac{l_j}{\sum_{i=1}^{n} l_i} f_j(h_j)\right) \\ &= \bigcup_{\gamma_1 \in h_1, \cdots, \gamma_n \in h_n}\left\{1 - \prod_{j=1}^{n}(1 - f(\gamma_j))^{\frac{l_j}{\sum_{i=1}^{n} l_i}}\right\} \end{aligned}$$

where $f_i : [0, 1] \to [0, 1]$ $(i = 1, 2 \cdots, n)$ is a strictly continuous monotonic function.

Obviously, if $f_i(x) = x$ $(i = 1, 2 \cdots, n)$, then the HFPMWA operator can be reduced to the HFPWA operator.

4. An approach to multi-attribute decision making with hesitant fuzzy information

In this section, we utilize the proposed hesitant fuzzy prioritized aggregation operators to solve group decision-making problems under hesitant fuzzy environment. In a group decision-making problem, suppose $X = \{x_1, x_2, \cdots, x_m\}$ is a set of alternatives, $G = (G_1, G_2, \cdots, G_n)$ is a collection of attributes and there is a prioritized relationship between these attributes expressed by the linear ordering $G_1 \succ G_2 \succ \cdots \succ G_n$, indicating that attribute G_j has a higher priority to G_k, if $j < k$. If decision makers provide all the possible evaluated values under the attribute G_j for the alternative x_i with anonymity, these values can be considered as a hesitant fuzzy element h_{ij}. In the case where two decision makers provide the same value, then the value emerges only once in h_{ij}. So we can construct the hesitant fuzzy decision matrix $H = (h_{ij})_{m\times n}$, where h_{ij} $(i = 1, 2, \cdots, m, j = 1, 2, \cdots, n)$ are in the form of HFEs. Based on the proposed operators, we give a method for the group decision-making problem, which involves the following steps:

Step 1. Calculate the weights ω_{ij} of h_{ij} $(i = 1, 2, \cdots, m, j = 1, 2, \cdots, n)$ as follow: for any $i(i = 1, 2, \cdots, m)$,

$$T_{i1} = \{1\},$$
$$T_{ij} = T_{i,j-1} \cap h_{i,j-1}, \quad j = 2, \cdots, n \tag{4.15}$$

$$l_{ij} = \prod_{k=1}^{j} s(T_{ik}), \quad j = 1, 2, \cdots, n \tag{4.16}$$

$$\omega_{ij} = \frac{l_{ij}}{\sum_{j=1}^{n} l_{ij}}, \quad j = 1, 2, \cdots, n \tag{4.17}$$

Step 2. Aggregate the hesitant fuzzy values h_{ij} $(j = 1, 2, \cdots, n)$ by using a hesitant fuzzy prioritized aggregation operator, denoted by Θ, then

$$h_i = \Theta(h_{i1}, h_{i2}, \cdots, h_{in}), i = 1, 2, \cdots, m.$$

Θ can be the GHFPWA operator, the GHFPOWA operator, the QHFPWA operator, the QHFPOWA operator, or the HFPMWA operator.

Step 3. Calculate the scores $s(h_i)$ $(i = 1, 2, \cdots, m)$ and the variance values $var(h_i)$ $(i = 1, 2, \cdots, m)$.

Step 4. Rank the overall hesitant fuzzy preference values h_i $(i = 1, 2, \cdots, m)$ by Definition 2.3, and select the best alternative(s).

5. Practical example

The school of management in a Chinese university wants to strengthen academic education, promote the building of teaching body. It is necessary to recruit oversea outstanding faculties (adapted from [30]). This introduction has been raised great attention from the school, university president, dean of management school and human resource officer and sets up the panel of decision makers which will take the whole responsibility for this introduction. They have made strict evaluation for 5 candidates x_i $(i = 1, 2, 3, 4, 5)$ from four aspects, namely morality G_1, research capability G_2, teaching skill G_3, education background G_4. In addition, this program is in strict accordance with the principle of combine ability with political integrity. The prioritization relationship between attributes is shown as: $G_1 \succ G_2 \succ G_3 \succ G_4$, The five candidates x_i $(i = 1, 2, 3, 4, 5)$ are to be evaluated by the three decision makers under the above four attributes with anonymity, and construct the hesitant decision matrix $H = (h_{ij})_{5\times 4}$, which is shown in Table 1.

We now use the method in Section 4 to select the best candidate(s). The main step is described as following:

Step 1. Utilize Equations (4.15)–(4.17) to calculate the values of ω_{ij} $(i = 1, 2, \cdots, 5, j = 1, 2, 3, 4)$ as follow:

$$(\omega_{ij})_{5\times 4} = \begin{bmatrix} 0.5065 & 0.2701 & 0.1441 & 0.0793 \\ 0.4391 & 0.3074 & 0.169 & 0.0845 \\ 0.4963 & 0.3474 & 0.1158 & 0.0405 \\ 0.5021 & 0.3013 & 0.1356 & 0.061 \\ 0.4398 & 0.2858 & 0.1715 & 0.1029 \end{bmatrix}$$

Step 2. Aggregate the hesitant fuzzy values h_{ij} $(j = 1, 2, 3, 4)$ by using the (GHFPWA) operator to derive the overall hesitant fuzzy values h_i $(i = 1, 2, 3, 4, 5)$ of the candidates x_i. If $\lambda = 1$ and alternative x_1 is taken for an example, then we have$h_1 = \text{GHFPWA}_\lambda(h_{11}, h_{12}, h_{13}, h_{14}) = \text{GHFPWA}_\lambda(\{0.4, 0.5, 0.7\}, \{0.5, 0.8\}, \{0.6, 0.7, 0.9\},$

Table 1
Hesitant fuzzy decision matrix H

	G_1	G_2	G_3	G_4
x_1	{0.4, 0.5, 0.7}	{0.5, 0.8}	{0.6, 0.7, 0.9}	{0.5, 0.6}
x_2	{0.6, 0.7, 0.8}	{0.5, 0.6}	{0.4, 0.6, 0.7}	{0.4, 0.5}
x_3	{0.6, 0.8}	{0.2, 0.3, 0.5}	{0.4, 0.6}	{0.5, 0.7}
x_4	{0.5, 0.6, 0.7}	{0.4, 0.5}	{0.8, 0.9}	{0.3, 0.4, 0.5}
x_5	{0.6, 0.7}	{0.5, 0.7}	{0.7, 0.8}	{0.2, 0.3, 0.4}

Table 2
The rankings of alternatives with different values of λ

λ	$s(h_1)$	$s(h_2)$	$s(h_3)$	$s(h_4)$	$s(h_5)$
$\frac{1}{2}$	0.6172	0.6232	0.5753	0.6045	0.6323
		$x_5 \succ x_2 \succ x_1 \succ x_4 \succ x_3$			
1	0.6216	0.6258	0.5843	0.6102	0.636
		$x_5 \succ x_2 \succ x_1 \succ x_4 \succ x_3$			
2	0.6308	0.631	0.6009	0.6228	0.6432
		$x_5 \succ x_2 \succ x_1 \succ x_4 \succ x_3$			
3	0.6406	0.6363	0.6149	0.6362	0.6498
		$x_5 \succ x_1 \succ x_2 \succ x_4 \succ x_3$			
5	0.6596	0.6463	0.6355	0.6633	0.661
		$x_4 \succ x_5 \succ x_1 \succ x_2 \succ x_3$			
7	0.6763	0.6549	0.6492	0.688	0.6704
		$x_4 \succ x_1 \succ x_5 \succ x_2 \succ x_3$			

$\{0.5, 0.6\}) =$

$$\bigcup_{\gamma_{11}\in h_{11},\gamma_{12}\in h_{12},\gamma_{13}\in h_{13},\gamma_{14}\in h_{14}} \left\{1 - \prod_{j=1}^{4}(1-\gamma_{1j})^{w_{1j}}\right\}$$

$= \{0.469, 0.4783, 0.4905, 0.4995, 0.5158, 0.5243, 0.5355, 0.5436, 0.5651, 0.5727, 0.5854, 0.5927, 0.6022, 0.6035, 0.6092, 0.6104, 0.622, 0.6262, 0.6286, 0.6328, 0.6373, 0.6414, 0.6437, 0.6477, 0.6605, 0.6664, 0.6904, 0.6939, 0.6959, 0.6992, 0.7082, 0.7133, 0.72, 0.7249, 0.961, 0.7652\}$. The obtained HFEs h_2, h_3, h_4, h_5 are omitted.

Step 3. Calculate the scores $s(h_i)$ $(i = 1, 2, 3, 4, 5)$ of the overall hesitant fuzzy values h_i $(i = 1, 2, 3, 4, 5)$:

$s(h_1) = 0.6216$, $s(h_2) = 0.6258$, $s(h_3) = 0.5843$, $s(h_4) = 0.6102$, $s(h_5) = 0.636$.

Since $s(h_5) > s(h_2) > s(h_1) > s(h_4) > s(h_3)$, the ranking of the candidates x_i $(i = 1, 2, 3, 4, 5)$ is $x_5 \succ x_2 \succ x_1 \succ x_4 \succ x_3$. Thus the most desirable candidate is x_5.

If the parameter λ in the GHFPWA operator takes different values, then we can make further analysis on the ranking of alternatives, which is followed in Table 2.

From Table 2, we find that the larger λ is, the greater the score values of the overall hesitant fuzzy values will be. When $\lambda \geq 5$, the ranking results of x_1,x_2,x_3,x_4,x_5 will change. Analogously, we can perform the same analysis by adopting the GHFPOWA operator, the QHFPWA operator or the QHFPOWA operator.

Now we compare the result by using the GHFPWA operator (with $\lambda = 1$) with that in [16]. Wei [16] proposed a hesitant fuzzy prioritized weighted average operator, denoted by HFPWA_{Wei}. The difference with the proposed GHFPWA operator with $\lambda = 1$ is the method deriving the weights of attributes.

From the hesitant fuzzy decision matrix in Table 1, the weight matrix can be calculated by using Wei's method [16] as follows:

$$W = \begin{bmatrix} 0.4686 & 0.2498 & 0.1626 & 0.119 \\ 0.4342 & 0.304 & 0.1672 & 0.0947 \\ 0.4878 & 0.3415 & 0.1137 & 0.0571 \\ 0.4762 & 0.2857 & 0.1286 & 0.1095 \\ 0.4286 & 0.2786 & 0.1672 & 0.1256 \end{bmatrix}$$

Then the scores of the overall hesitant fuzzy values of the candidates x_i $(i = 1, 2, 3, 4, 5)$ can be shown as follows: $s(h_1) = 0.6241$, $s(h_2) = 0.6242$, $s(h_3) = 0.585$, $s(h_4) = 0.602$, $s(h_5) = 0.6302$. Thus the ranking of all the candidates is $x_5 \succ x_2 \succ x_1 \succ x_4 \succ x_3$, and the most desirable candidate is x_5, which is the same with the result in this paper.

From the analysis above, the decision-making method proposed in this paper has two main advantage: (1) the process of deriving the weights of attributes can avoid information loss as far as possible and can highlight the prioritization between attributes more better; (2) the decision-making method in this paper can provide more aggregation ways for decision information through the variety of aggregation functions and the change of parameter λ. Moreover, more flexible selections can be provided to decision makers by changing the strictly continuous monotonic function $(f : [0, 1] \rightarrow [0, 1])$ in QHFPWA, QHFPOWA, HFPMWA.

6. Conclusion

In this paper, we investigated the hesitant fuzzy MADM problems in which the attributes are in different priority level. Based on the idea of prioritized aggregation operator [2, 5, 21] and motivated by the idea of hesitant fuzzy prioritized aggregation operator [16], we have proposed a new method to determine the weighting vectors of attributes associated with the prioritized relationship of the aggregated arguments. The method proposed in this paper can avoid fuzzy information loss as far as possible. Based on the proposed method, we have defined the HFPWA operator, the GHFPWA operator, the QHFPWA operator and the HFPMWA operator. We also proposed some extended hesitant fuzzy prioritized OWA operators in order to consider both the prioritization relationship and the aggregation requirements of decision makers. We have applied these proposed prioritized aggregation operators to develop a MADM method that take into account prioritization among attributes. Finally, an example has been given to illustrate the effectiveness of decision-making method. It is worth noticing that the results of this paper can be extended to the hesitant fuzzy linguistic environment and our future work may focus on that.

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